MTHSC 102 Section 1.2 – Linear Functions and Models

Kevin James

Definition

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Note

When we graph the output of a linear function against its input the points lie along a lline.

Consider a newspaper delivery team that makes weekly deliveries of newspapers and devotes their Saturday mornings to selling new subscriptions. Suppose that the number of customers in week during week w is given by the following table where w is the number of weeks since the beginning of business.

Weeks	Number of customers
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Graph the above data.

Give a linear model describing this data.

Fact

All linear functions have the form $f(x) = ax + b$ *where* a *and* b *are constants.*

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Fact

Given any two data points for a linear function

input	*output*
x1	*y1*
x2	*y2*

we can compute the slope as follows

$$slope = \frac{y2 - y1}{x2 - x1}.$$

Example

Compute the slope of the linear function from the previous example.

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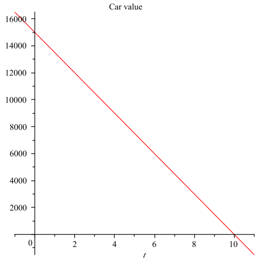
Note

The slope of a graph at a particular point is a measure of how quickly the graph is changing at that point. This measure is called the rate of change of the function at that point.

Suppose that the resale value of a used car is represented graphically by the following graph.



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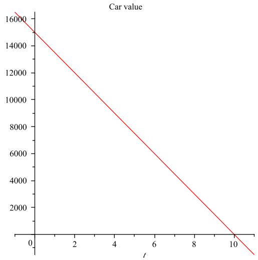

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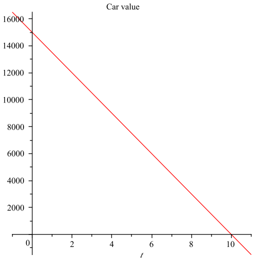

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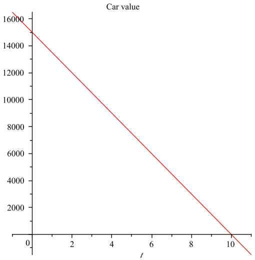

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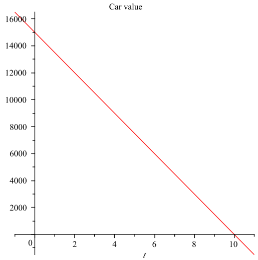

1. Locate the point where the input is 0 and the point where the output is 0.
2. Calculate the slope of the graph.
3. Does the slope depend on the direction in which it is calculated?
4. Interpret the slope.
5. Write a linear model for the graph.

Definition

Suppose we are considering the graph of a linear model or function.

1. If the graph of the function rises as we move to the right (-i.e. if the slope is positive) then we say that the function is increasing.
2. If the graph of the function falls as we move to the right (-i.e. if the slope is negative) then we say that the function is decreasing.
3. If the function is neither increasing nor decreasing (-i.e. if the slope is 0) then we say that the function is constant.

Suppose that the taxes paid by a certain business during the years from 1999 to 2004 are given by the following table.

Year	1999	2000	2001	2002	2003	2004
Tax	\$ 2503	\$ 3008	\$ 3513	\$ 4018	\$ 4523	\$ 5028

Consider the first differences (-i.e. the differences in output corresponding to successive equally spaced inputs),

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Thus the data can be modeled by a linear function.

The slope is 505 dollars per year.

So letting t denote years past 1999, we have an intercept of $2503 and the linear model is

$$T(t) = 505t + 2503 \text{ dollars},$$

where T denotes the amount of taxes paid in the year t years after 1999.

Note

Our choice to let t denote years past 1999 was arbitrary. This is called aligning our data. A good choice of data alignment will often result in smaller constants in our models. For example, if the model is linear, alignment will not change the slope by will change the vertical axis intercept.

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If we use our model to predict the output for an input lying within this range this process is called interpolation.
If we use our model to predict the output for an input lying outside this range, this process is called extrapolation.
Predictions involving extrapolation should be viewed with skepticism and caution because we may not have any reason to believe that the real-world process which we have modeled will continue to behave in the same way outside of the range that we have observed.

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For example, if the table of data in the previous example had been

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Tax	\$2502		\$3008		\$3512		\$4019		\$4524		\$5030
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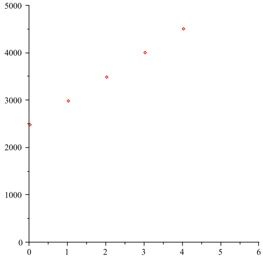
Thus our data is not linear, but is nearly linear.

Here is a scatter plot of the data from the previous page.



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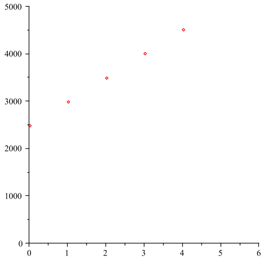
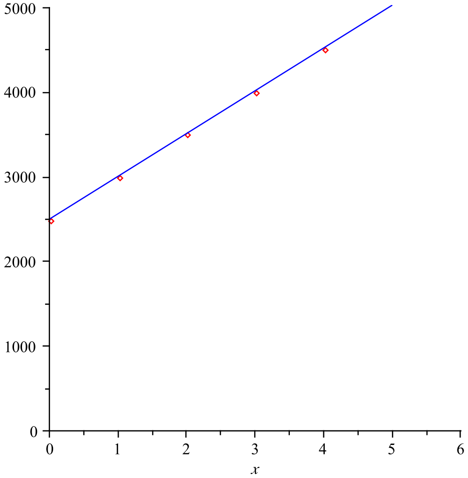


We can use our calculators to find the line that is the best fit to the data, namely

$$T(t) = 505.7714286t + 2502.571429$$

Let's graph the line of best fit over our scatter plot.



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MODELS Always use unrounded models for computations.

COMPUTATION Postpone rounding until the final result is computed.

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Make sure that all of your models have the following crucial elements.

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4. An indication of the interval of input values over which the model is valid.