

MTHSC 102 SECTION 1.3 – EXPONENTIAL AND LOGARITHMIC FUNCTIONS AND MODELS

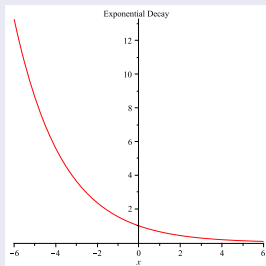
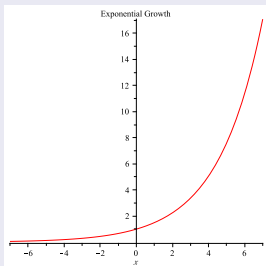
Kevin James

DEFINITION

An exponential function has constant percentage change or is the result of repeated multiplication.

It has an equation of the form $f(x) = ab^x$ where $a \neq 0$ is the initial value (-i.e. the value of $f(0)$) and $b > 0$ is the constant multiplier. The percentage change is $(b - 1) \times 100\%$.

The graphs of exponential functions with $a > 0$ has one of the following forms



EXAMPLE

During 2001 iPod sales were 0.14 million units and until 2005 sales increased approximately 260% each year.

- 1 Why is an exponential model appropriate for iPod sales?
- 2 Find a model for iPod sales.
- 3 According to the model what were the 2006 iPod sales?

EXAMPLE (MODELING FROM DATA)

The following data represents the dwindling population in a mill town t years after the closing of the mill.

Year	0	1	2	3	4	5	6
Population	7290	5978	4902	4020	3296	2703	2216

Consider the first differences

Pop.	7290	5978	4902	4020	3296	2703	2216
Diff.		-1312	-1076	-882	-724	-593	-487
%		-17.997	-17.999	-17.993	-18.010	-17.992	-18.017

Since the percentage differences are nearly constant, the use of an exponential model is appropriate.

Using our calculators we have the model

$$P(t) = 7290.366(0.819995)^t \text{ people.}$$

DOUBLING TIME AND HALF LIFE

DEFINITION

- 1 Doubling Time is the amount of time it takes for the output of an increasing exponential function to double.
- 2 Half Life is the amount of time it takes for the output of a decreasing exponential function to decrease by half.

EXAMPLE

Suppose that the amount of a certain drug in a patient's system has a half life of 2 hours.

- 1 Write a model for the amount of this drug left in a person's body if the initial dose is 100mg.
- 2 If it is safe to take another dose of this drug once the amount in the body is less than 1mg, when should another dose be taken?

NOTE

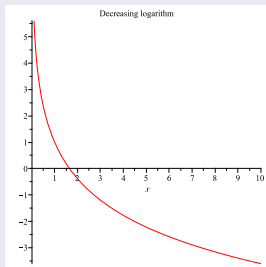
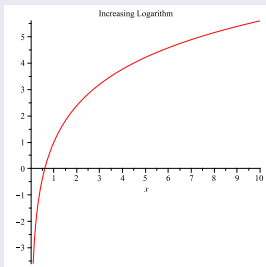
A good choice of alignment of input data may produce simpler models. Graphically, alignment of the input data simply translates the graph horizontally.

DEFINITION

A logarithmic function has a vertical asymptote ($x = 0$) and continues to grow or decline as x becomes large but at increasingly slower rates.

A log model has an equation of the form $f(x) = a + b \ln(x)$, where $b \neq 0$.

The graphs of log models have one of the following forms.



NOTE

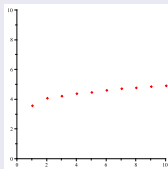
Logarithms do NOT have horizontal asymptotes.

EXAMPLE (FINDING A LOG MODEL)

An international investment fund manager models bond rates of countries. He uses the following data

Time to Maturity	1	2	3	4	5	6	7	8	9	10
Rate	3.60	4.10	4.25	4.40	4.50	4.65	4.75	4.8	4.9	4.95

Here is a sketch of the data.



- 1 Find a log model for the data.
- 2 What does your model estimate as the rate for 20 year bonds?
for 30 year bonds?