MTHSC 102 Section 1.3 – Exponential and Logarithmic Functions and Models

Kevin James

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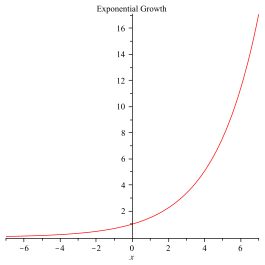
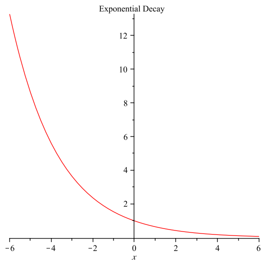
The percentage change is $(b-1) \times 100\%$.
The graphs of exponential functions with $a > 0$ has one of the following forms





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2. Find a model for iPod sales.
3. According to the model what were the 2006 iPod sales?

Example (Modeling from Data)

The following data represents the dwindling population in a mill town t years after the closing of the mill.

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Using our calculators we have the model

$P(t) = 7290.366(0.819995)^t$ people.

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Example

Suppose that the amount of a certain drug in a patient's system has a half life of 2 hours.

1. Write a model for the amount of this drug left in a person's body if the initial dose is 100mg.
2. If it is save to take another dose of this drug once the amount in the body is less than 1mg, when should another dose be taken?

Note

A good choice of alignment of input data may produce simpler models. Graphically, alignment of the input data simply translates the graph horizontally.

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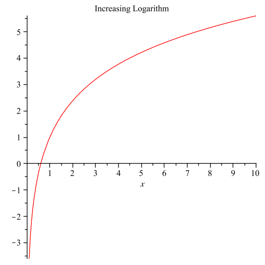

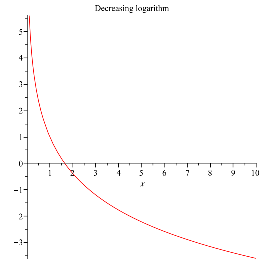

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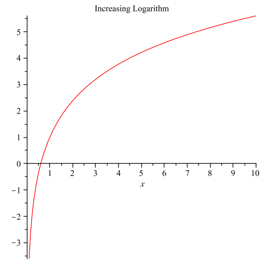

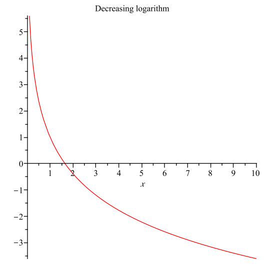

Note

Logarithms do NOT have horizontal asymptotes.

Example (Finding a log model)

An international investment fund manager models bond rates of countries. He uses the following data

Time to Maturity	1	2	3	4	5	6	7	8	9	10
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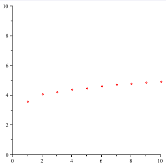
Here is a sketch of the data.



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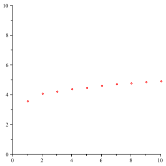

1. Find a log model for the data.
2. What does your model estimate as the rate for 20 year bonds? for 30 year bonds?