MTHSC 102 Section 1.5 – Polynomial Functions and Models

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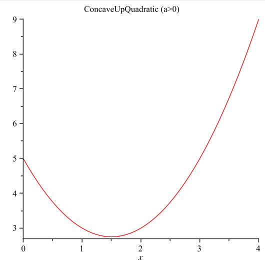
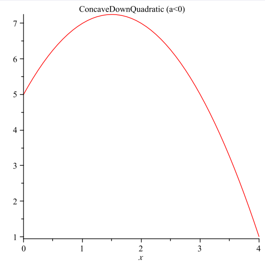
Definition

A quadratic function is a function whose second differences are constant. It achieves either a local max or a local min and has no inflection points.

A quadratic model has an equation of the form $f(x) = ax^2 + bx + c$, where $a \neq 0$ is a constant which determines the concavity of the model, and b and c are constants.

A quadratic function has one of the following forms.





If $a > 0$, then $\lim_{x\to\pm\infty}(ax^2 + bx + c) = \infty$.

If $a < 0$, then $\lim_{x\to\pm\infty}(ax^2 + bx + c) = -\infty$.

Example

A roofing company records the number of jobs completed each month. The data is recorded below.

Month	Jan	Feb	Mar	Apr	May	Jun
Jobs	90	91	101	120	148	185

We compute the first and second differences.

Jobs	90	91	102	123	154	195
1st		1	11	21	31	41
2nd			10	10	10	10

Since the 2nd differences are constant, a quadratic function will best fit the data.

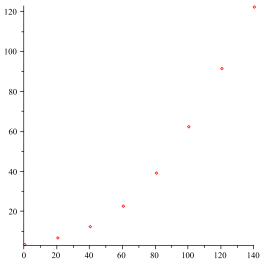
Use your calculator to find a quadratic model.

Example

The following table shows the population of the contiguous United States for selected years between 1790 and 1930.

Year	1790	1810	1830	1850	1870	1890	1910	1930
Pop. (millions)	3.929	7.240	12.866	23.192	39.818	62.948	91.972	122.775

The following is a scatter plot of the data.

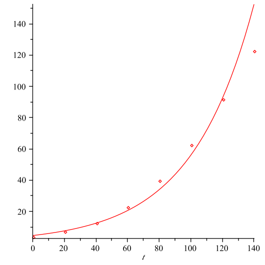


Example Continued ...

Since the data records population and the scatter plot does not seem to have a local min, we might try an exponential model, such as

$$P(t) = 4.558(1.285)^t \text{ million people,}$$

where t is the number of **decades** since 1790. Graphing this model against our data, we have



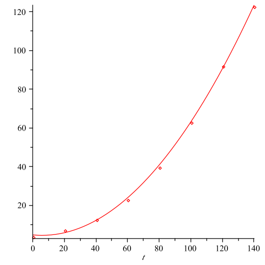
Not such a good fit.

Example Continued ...

We could next try a quadratic model such as

$$P(t) = 0.66t^2 - 0.77t + 4.31 \text{ million people},$$

where t is the number of decades since 1790. Graphing this model against the data, we have



This is a much better fit in our data range.

Definition

A cubic function is a function whose third differences are constant. It can achieve either a local max and a local min or neither. It has one inflection point.

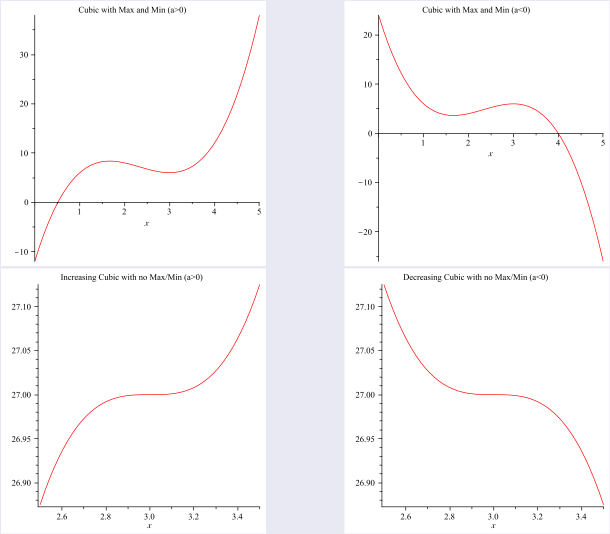
A cubic model has an equation of the form

$$f(x) = ax^3 + bx^2 + cx + d,$$

where $a \neq 0$ is a constant and b, c and d are constants.

Note

A cubic function has one of the following forms.



Fact (End behavior)

If $a > 0$, *then*

$$\lim_{x\to-\infty}(ax^2+bx+c) = -\infty, \quad \text{and} \quad \lim_{x\to\infty}(ax^3+bx^x+cx+d) = \infty.$$

If $a < 0$, *then*

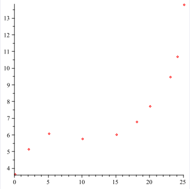
$$\lim_{x\to-\infty}(ax^2+bx+c) = \infty, \quad \text{and} \quad \lim_{x\to\infty}(ax^3+bx^x+cx+d) = -\infty.$$

Example

The average price in dollars per 1000 cubic feet of natural gas for residential use in the US for selected years from 1980 through 2005 is given in the following table.

Year	1980	1982	1985	1990	1995	1998	2000	2003	2004	2005
Price	3.68	5.17	6.12	5.80	6.06	6.82	7.76	9.52	10.74	13.84

Plotting this data, we have the following scatter plot.



1. Find a cubic model for this data.
2. Would it be wise to use this model to predict future gas prices?
3. Estimate the price in 1993.

Choosing a Model

1. Examine a scatter plot of your data.
    1. If the plot appears to be near a straight line, then try a linear model.
    2. If the data appears to lie on a curve and there is no inflection point try an exponential, log or quadratic model. Note that the exponential and quadratic models are very different from the log model.
    3. If there appears to be an inflection point try a cubic or logistic model.
2. Look at the fit of the (at most 2) possible models from the first step.
3. Look at the end behavior. Perhaps you can discern between models not separated above by end behavior.
4. Consider that thee may be two equally good choices of model.