MTHSC 102 Section 1.5 – Polynomial Functions and Models

Kevin James

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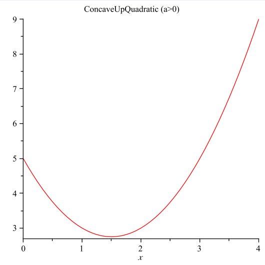

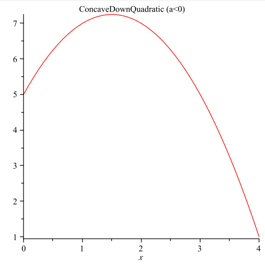

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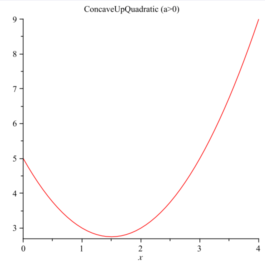

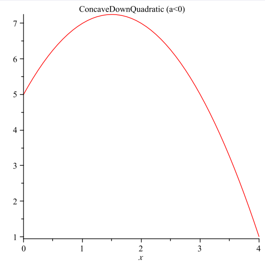

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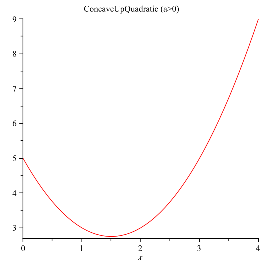

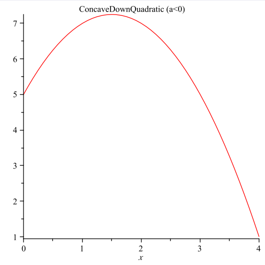

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Example

A roofing company records the number of jobs completed each month. The data is recorded below.

Month	Jan	Feb	Mar	Apr	May	Jun
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Use your calculator to find a quadratic model.

Example

The following table shows the population of the contiguous United States for selected years between 1790 and 1930.

Year	1790	1810	1830	1850	1870	1890	1910	1930
Pop. (millions)	3.929	7.240	12.866	23.192	39.818	62.948	91.972	122.775

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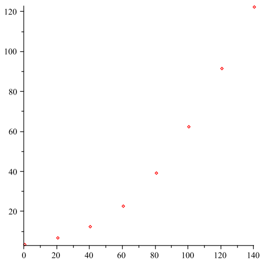
The following is a scatter plot of the data.



Example Continued ...

Since the data records population and the scatter plot does not seem to have a local min, we might try an exponential model, such as

$$P(t) = 4.558(1.285)^t \text{ million people,}$$

where t is the number of **decades** since 1790.

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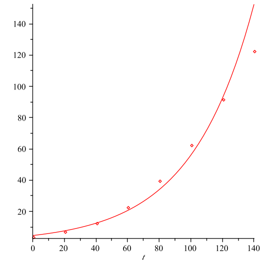

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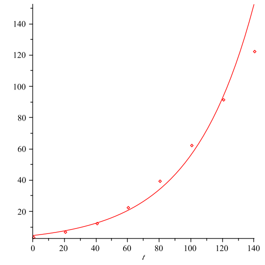

Not such a good fit.

Example Continued ...

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$$P(t) = 0.66t^2 - 0.77t + 4.31 \text{ million people,}$$

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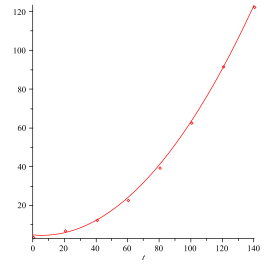

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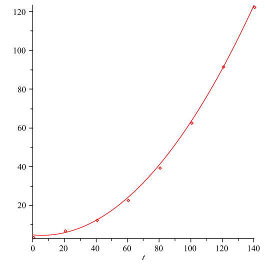

This is a much better fit in our data range.

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A cubic model has an equation of the form

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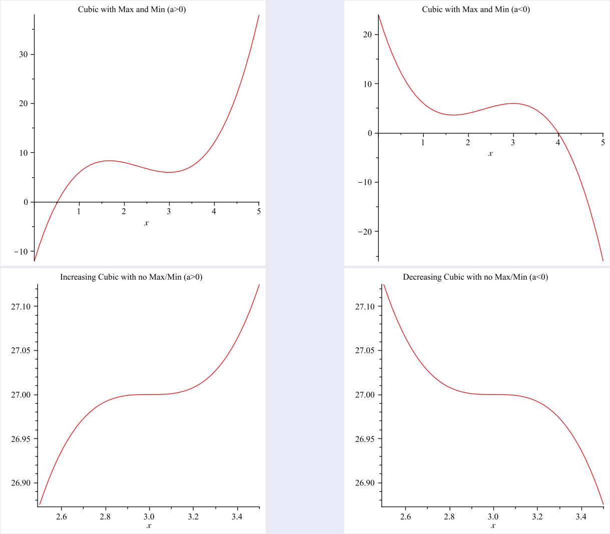
where $a \neq 0$ is a constant and b, c and d are constants.

Note

A cubic function has one of the following forms.



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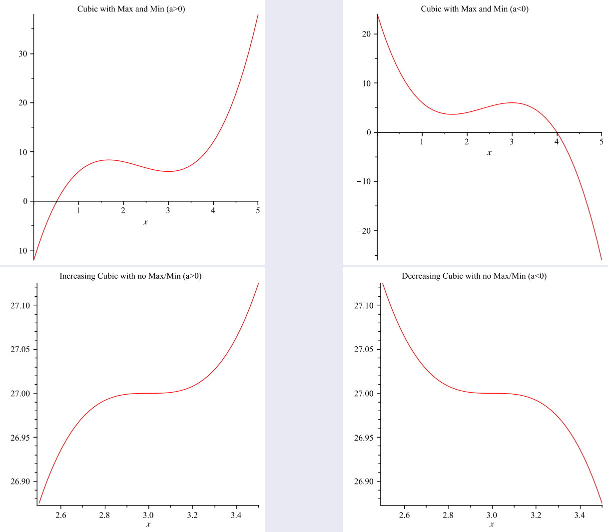

Fact (End behavior)

If $a > 0$, then

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The average price in dollars per 1000 cubic feet of natural gas for residential use in the US for selected years from 1980 through 2005 is given in the following table.

Year	1980	1982	1985	1990	1995	1998	2000	2003	2004	2005
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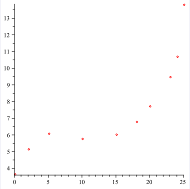

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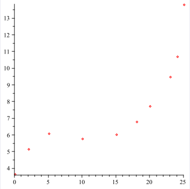

1. Find a cubic model for this data.
2. Would it be wise to use this model to predict future gas prices?
3. Estimate the price in 1993.

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4. Consider that thee may be two equally good choices of model.