MTHSC 102 Section 2.1 – Change, Percentage change, Average Rate of Change

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Definition

Suppose that an quantity (typically the output of a function) changes from m to n over an (input) interval from a to b. We define the following descriptions of this change.

- Change $= n - m$.
- Percentage change $= \frac{\text{change}}{\textit{firstvalue}} \cdot 100\% = \frac{n-m}{m} \cdot 100\%$.
- Average Rate of Change $= \frac{\text{change}}{\text{length of interval}} = \frac{n-m}{b-a}$.

Interpretation of Descriptions of Change

When discussing change over an interval, be sure to include the following information.

1. Specify the input interval.
2. Specify the quantity that is changing.
3. Indicate whether the change is a decrease or increase.
4. Give the numerical answer labeled with units.

Description	**Units**
change	output unit of measure
percentage change	percent
average rate of change	output unit of measure per single input unit of measure

Example

The following table records the temperature at a particular location during a certain day.

Time	7am	8am	9am	10am	11am	noon	1pm	2pm	3pm	4pm	5pm
Temp	49	58	66	72	76	79	80	80	78	74	69

1. Describe the change in temperature from 7am to noon.
2. Describe the percentage change in temperature between 11am and 4pm.
3. Describe the average change in temperature between 9am and 5pm.

Describing Change from a Graph

In order to describe the change in a function $f(x)$ over an interval $[a, b]$, we draw the secant line from $(a, f(a))$ to $(b, f(b))$.

- change $= f(b) - f(a)$.
- average rate of change $= \frac{f(b)-f(a)}{b-a} =$ slope of secant line.
- percentage change $= \frac{f(b)-f(a)}{f(a} \cdot 100\%$.

Note

The above information could be read from a graph or calculated from an explicit description of the function f.

Example

A model for temperature on a certain day in a certain location is

$$T(t) = -t^2 + 2t + 80^\circ F,$$

where t is the number of hours after noon.
Calculate the percentage change and average rate of change between 11:30am and 5pm.

Compound Interest Formula

The amount accumulated in an account after t years when P dollars are initially invested at an annual interest rate of $100r\%$ compounded n times per year is

$$A(t) = P\left(1 + \frac{r}{n}\right)^{nt} \text{ dollars.}$$

The annula percentage rate (APR) is the percentage $100r\%$ and the percentage change of the amount accumulated over one compounding period is $100\frac{r}{n}\%$.
The effective rate or annual percentage yield (APY) is the percentage change of the account accumulated over one year.

Example

Suppose that you are 25 years old and have $10,000 to invest for retirement.

1. What APR compounded monthly is needed for your money to grow to $1,000,000 in 40 years?
2. What is the APY for this investment?
3. What are the annual and monthly percentage changes for this investment?

Continuously Compounded Interest Formula

The amount accumulated in an account after t years when P dollars are invested at a nominal rate (APR) of $100r\%$ compounded continuously is

$$A(t) = Pe^{rt} \text{ dollars.}$$

Example

An investment that has interested compounded continuously has an APY of 9.2%. What is the APR?