

MTHSC 102 SECTION 2.2 – INSTANTANEOUS RATES OF CHANGE

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DEFINITION

The instantaneous rate of change at a point on a curve is the slope of the graph at that point.

LOCAL LINEARITY

If we look closely enough near any point P on a smooth curve, the curve will be very close to the line tangent to the curve at P . That is, the tangent line is a very good approximation of the curve near P .

DEFINITION

The slope of a smooth graph at a point P is the slope of the line which is tangent to the graph at P .

LINE TANGENT TO A SMOOTH CURVE

The tangent line at a point Q on a smooth continuous graph is the limiting position of the secant lines between point Q and a point P as P approaches Q along the graph (provided the limiting position exists).

GENERAL RULE FOR TANGENT LINES

Lines tangent to a smooth nonlinear curve typically lie on one side or the other of the graph. Tangent lines only cross the graph if the point is an inflection point.

EXISTENCE OF INSTANTANEOUS RATE OF CHANGE

DEFINITION

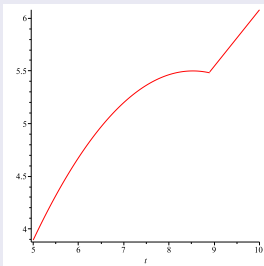
A piecewise continuous function is a function that is continuous over different intervals but has a break point. It is often defined by different equations over different intervals.

EXAMPLE

For example the function

$$f(x) = \begin{cases} -0.129t^2 + 2.25 - 3.88 & \text{when } 5 \leq t \leq 9, \\ 0.536t + 0.72 & \text{when } 9 \leq t \leq 10. \end{cases}$$

has the following graph.



At the point $(9, f(9))$, the tangent line is not well defined.

DEFINITION

We call a point P on the graph of a continuous function f a sharp point when the secant lines joining P to close points on either side of it have different limiting positions.

FACT

The line tangent to the graph of a function at a point P is defined only when the graph is smooth and continuous at the point P .