MTHSC 102 Section 2.3 – Derivative Notation and Numerical Estimates

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Average vs. Instantaneous Rates of Change

Average Rates of Change	Instantaneous Rates of Change
Measures the rate of change of a quantity over an interval	Measures the rate of change of a quantity at a point
Slope of a secant line	Slope of the tangent line
Requires data points or a continuous curve to calculate	Requires a continuous smooth curve to calculate

Equivalent Terminology

All of the following phrases mean the same.

- Instantaneous rate of change
- rate of change
- slope of the curve
- slope of the tangent line
- **derivative**

Derivative Notation

We have several notations for the derivative of $f(t)$ with respect to t, namely $\frac{df}{dt}$, $f'(t)$, $\frac{d}{dt}[f(t)]$.
Note that here f is the output variable (or function) and t is the input variable.

Interpreting Derivatives

When discussing instantaneous rate or change at a point (or the derivative of a function at a point), be sure to include the following information.

1. Specify the input value.
2. Specify the quantity that is changing.
3. Indicate whether the change is a decrease or increase.
4. Give the numerical answer labeled with proper units.
5. The units for the derivative should be the output units per one input unit (as for average rate of change).

Definition

We define the percent rate of change of a function as follows.

$$\text{Percent Rate of Change at } P = \frac{\text{rate of change at } P}{\text{value of the function at } P} \cdot 100\%$$

The units for this quantity are % per 1 input unit.
We could also write

$$\text{Percent Rate of Change at } P = \frac{\text{slope of the tangent line at } P}{\text{value of the function at } P} \cdot 100\%$$

Note

The line secant to $y = f(x)$ and passing through $P = (a, f(a))$ and $Q = (b, f(b))$ has slope

$$\text{slope} = \frac{f(b) - f(a)}{b - a}$$

Estimating the slope of the tangent line

1. Calculate the slope of the secant line passing through P and several nearby points $Q_i = (b_i, f(b_i))$ $(i = 1, 2, 3, \dots)$ to the left of P.
2. Does the slope seem to be getting close to some value as Q_i approaches P? If so, what value?
3. Calculate the slope of the secant line passing through P and several nearby points $Q_i = (b_i, f(b_i))$ $(i = 1, 2, 3, \dots)$ to the right of P.
4. Does the slope seem to be getting close to some value as Q_i approaches P? If so, what value?
5. If you obtained values for 2 and 4 and if they are the same, their common value is a good estimate for the slope of the tangent line.

Example

A company invests \$32 billiion of its assets electronically in the global market, resulting in an investment with continuous compounding at 12% per year APY.
Note that the value of such an investment after t years is given by $A(t) = 32 \cdot (1.12)^t$ billion dollars.

1. How rapidly is the investment growing at the beginning of the fifth year (-i.e. $t = 4$)?
2. At what percentage rate of change is this investment growing?