MTHSC 102 Section 2.3 – Derivative Notation and Numerical Estimates

Kevin James

Average vs. Instantaneous Rates of Change

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- **derivative**

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5. The units for the derivative should be the output units per one input unit (as for average rate of change).

Definition

We define the percent rate of change of a function as follows.

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We could also write

$$\text{Percent Rate of Change at } P = \frac{\text{slope of the tangent line at } P}{\text{value of the function at } P} \cdot 100\%$$

Note

The line secant to $y = f(x)$ and passing through $P = (a, f(a))$ and $Q = (b, f(b))$ has slope

$$\text{slope} = \frac{f(b) - f(a)}{b - a}$$

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Estimating the slope of the tangent line

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5. If you obtained values for 2 and 4 and if they are the same, their common value is a good estimate for the slope of the tangent line.

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1. How rapidly is the investment growing at the beginning of the fifth year (-i.e. $t = 4$)?
2. At what percentage rate of change is this investment growing?