MTHSC 102 Section 2.4 – Algebraically Finding Slopes

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Recall

1. $\lim_{x \to a^-} f(x)$ denotes the value that $f(x)$ approaches as x approaches (but is not equal to) a from the left.
2. $\lim_{x \to a^+} f(x)$ denotes the value that $f(x)$ approaches as x approaches (but is not equal to) a from the right.
3. $\lim_{x \to a} f(x)$ denotes the common value of the previous two **ONLY** when they both exist and they are the same.

Example

Consider the function $f(x) = \frac{x^2-9}{x-3}$. Estimate $\lim_{x \to 3^-} f(x)$, $\lim_{x \to 3^+} f(x)$ and $\lim_{x \to 3} f(x)$ numerically.

Example

Consider the function $g(x) = \frac{5x}{4x-12}$. Estimate $\lim_{x \to 3^-} g(x)$, $\lim_{x \to 3^+} g(x)$ and $\lim_{x \to 3} g(x)$ numerically.

Definition

A function f is <u>continuous</u> on an interval (a, b) if it is defined at every point in the interval and there are no breaks or jumps in the function output.
More precisely, the function must be defined on all of (a, b) and for all $c \in (a, b)$ it must be true that

$$\lim_{x \to c} f(x) = f(c).$$

Note

If the numerator and denominator of a rational function share a common factor, then the function obtained by algebraically canceling the common factor has all limits identical to those of the original function.

Example

Suppose that $f(x) = x^2 - 15x + 6$.

1. Find $\left.\dfrac{\mathrm{df}}{\mathrm{dx}}\right|_{x=2}$.
2. Find a formula for the derivative $f'(x)$ at an arbitrary point x.

Four Step Method to Find $f'(x)$

Given a function f, the equation for the derivative with respect to x can be found as follows.

1. Begin with a typical point $(x, f(x))$.
2. Choose a close point $(x+h, f(x+h))$.
3. Write a formula for the slope of the secant line between the two points
$$\text{slope} = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}.$$
4. Make sure to simplify the formula for slope as much as possible.
5. Evaluate
$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}.$$

This limiting value is the derivative formula at each input where the limit exists.

Derivative Formula

If $y = f(x)$, then the derivative $\frac{dy}{dx}$, $\frac{df}{dx}$, $f'(x)$ is given by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Example

Compute the derivative of the function $f(x) = 2\sqrt{x}$.