MTHSC 102 Section 2.4 – Algebraically Finding Slopes

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3. $\lim_{x \to a} f(x)$ denotes the common value of the previous two **ONLY** when they both exist and they are the same.

EXAMPLE

Consider the function $f(x) = \frac{x^2-9}{x-3}$. Estimate $\lim_{x\to 3^-} f(x)$, $\lim_{x\to 3^+} f(x)$ and $\lim_{x\to 3} f(x)$ numerically.

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Consider the function $g(x) = \frac{5x}{4x-12}$. Estimate $\lim_{x \to 3^-} g(x)$, $\lim_{x \to 3^+} g(x)$ and $\lim_{x \to 3} g(x)$ numerically.

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Note

If the numerator and denominator of a rational function share a common factor, then the function obtained by algebraically canceling the common factor has all limits identical to those of the original function.

Example

Suppose that $f(x) = x^2 - 15x + 6$.

1. Find $\left.\dfrac{\mathrm{df}}{\mathrm{dx}}\right|_{x=2}$.
2. Find a formula for the derivative $f'(x)$ at an arbitrary point x.

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3. Write a formula for the slope of the secant line between the two points
$$\text{slope} = \frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}.$$

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This limiting value is the derivative formula at each input where the limit exists.

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Example

Compute the derivative of the function $f(x) = 2\sqrt{x}$.