MTHSC 102 Section 3.1 – Drawing Rate of Change Graphs

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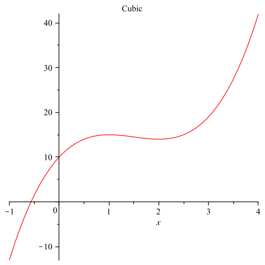
Definition

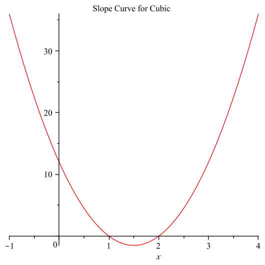
Suppose that $y = f(x)$ is a smooth continuous curve. The graph of $y = f'(x)$ is called the slope graph for this curve.

Example

The graph

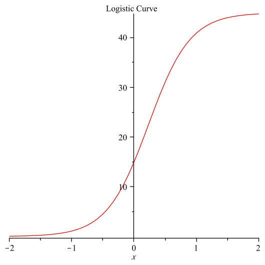


has the slope graph

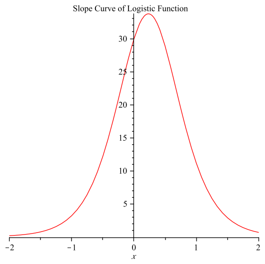


Example

Consider the logistic curve.



It has the slope graph



Example

1. Sketch a graph which is always increasing and whose slopes are always increasing.
2. Sketch a graph which is always increasing and whose slopes are always decreasing.
3. Sketch the slope graphs of these functions.

Sketching Slope Graphs

When sketching the slope graph of a function, the following information is very useful.

1. Points at which the tangent line is horizontal. These are the zeroes of the derivative.
2. Intervals over which the graph is increasing or decreasing. These are the intervals where the derivative $f'(x)$ is either positive or negative.
3. Points of inflection. These are locations of local extrema of the derivative.
4. Places where the graph is horizontal or appears to be leveling off. These will be places where the derivative is approaching zero.

Note

Given a graph we can draw a few tangent lines and estimate the slopes in order to plot some points on the graph of the derivative graph. That is, if the slope of the line tangent to $y = f(x)$ at a is m then $f'(a) = m$. Thus, the point (a, m) is on the graph of $f'(x)$.

Note

The value of $f'(x)$ will be defined at a if

1. The graph of $f(x)$ is undefined at a. That, is the graph of $y = f(x)$ has a hole above a or perhaps has a vertical asymptote $x = a$.
2. The graph of $f(x)$ is defined but not continuous at a. That, is the graph of $y = f(x)$ has a jump at a.
3. The graph of $f(x)$ is continuous but not smooth at a. That, is the graph of $y = f(x)$ has a sharp point above a.
4. The graph of $f(x)$ is continuous and smooth and has a vertical tangent line at $(a, f(a))$.