MTHSC 102 Section 3.4 – The Chain Rule

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The chain rule is a tool for evaluating the derivative of the composition of two functions for which we know the derivatives.

The Chain Rule (1st form)

Suppose that C is a function of p and that p is a function of t. Then we can consider C as a function of p by composing C and p and ask for the rate of change of C with respect to t.

$$\frac{\mathrm{dC}}{\mathrm{dt}} = \left(\frac{\mathrm{dC}}{\mathrm{dp}}\right)\left(\frac{\mathrm{dp}}{\mathrm{dt}}\right)$$

Example

Let $A(v)$ denote the average cost to produce a violin when v violins are produced and let $v(t)$ denote the number (in thousands) of violins produced t years after 2000.
Suppose that 10 thousand violins are produced in 2008 and that the average cost to produce a violin at that time is \$142.10.
Also suppose that in 2008 the production of violins is increasing by 100 violins per year and the average cost of production is decreasing by \$0.15 per violin.

1. Describe the meaning and give the value of each of the following in 2008.
    1. $v(t)$
    2. $v'(t)$
    3. $A(v)$
    4. $A'(v)$
2. Calculate the rate of change with respect to time of the average cost for violins in 2008.

The Chain Rule (2nd form)

If a function f can be expressed as the composition of two functions g and h, that is $f = h \circ g(x) = h(g(x))$, then

$$\frac{\mathrm{df}}{\mathrm{dx}} = f'(x) = h'(g(x)) \cdot g'(x).$$

Example

Write the derivatives with respect to x of the following functions.

1. $y = e^{x^2}$
2. $y = (x^3 + 2x^2 + 4)^{\frac{1}{2}}$
3. $y = \frac{3}{4-2x^2}$