

MTHSC 102 SECTION 3.4 – THE CHAIN RULE

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$$\frac{dC}{dt} = \left(\frac{dC}{dp} \right) \left(\frac{dp}{dt} \right)$$

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- 2 Calculate the rate of change with respect to time of the average cost for violins in 2008.

THE CHAIN RULE (2ND FORM)

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EXAMPLE

Write the derivatives with respect to x of the following functions.

① $y = e^{x^2}$

② $y = (x^3 + 2x^2 + 4)^{\frac{1}{2}}$

③ $y = \frac{3}{4-2x^2}$