

# MTHSC 102 SECTION 3.5 – THE PRODUCT RULE

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## EXAMPLE

Suppose that a certain University charges  $T(t)$  dollars for tuition in the year beginning  $t$  years after August of 2000. Suppose also that the enrollment for the same year at the same University is given by  $E(t)$ .

- 1 Give a formula for the total revenue from student tuition in year  $t$ .
- 2 Suppose that during the 2007-2008 school year, tuition was \$ 9,000 and that enrollment was 18,503 students. Suppose also that tuition was increasing at a rate of \$ 500 per year and that the enrollment at the University was decreasing at a rate of 50 students per year. What was the total revenue in during this year?
- 3 What was the rate of change in total revenue at this time?

## THEOREM (PRODUCT RULE)

If  $f(x) = g(x) \cdot h(x)$ , then

$$\frac{df}{dx} = \frac{dg}{dx}h(x) + g(x)\frac{dh}{dx}.$$

## EXAMPLE

In the previous example, revenue was given by  $R(t) = T(t) \cdot E(t)$ . So, the rate of change in revenue is given by

$$\frac{dR}{dt} = \frac{dT}{dt} \cdot E(t) + T(t) \cdot \frac{dE}{dt}.$$

## EXAMPLE

A music store has determined that when the price of a CD is  $x$  dollars, the number of CDs sold in a 4 week period can be modeled by

$$N(x) = 6250(0.929)^x \text{ CDs.}$$

- 1 What equation models the revenue generated by CD sales during a 4 week period when the price is  $x$  dollars per CD?
- 2 What is the rate of change in revenue when the price is \$ 10, \$ 12, \$ 15.

## THEOREM (QUOTIENT RULE)

Suppose that  $f(x) = \frac{g(x)}{h(x)}$ . Then the derivatives are related by

$$f'(x) = \frac{g'(x)h(x) - g(x)h'(x)}{[h(x)]^2}.$$

## NOTE

The quotient rule can be achieved by applying the product and chain rules to  $f(x) = g(x) \cdot [h(x)]^{-1}$ .