MTHSC 102 Section 4.2 – Relative and Absolute Extreme Points

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Definition (Extreme Points)

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1. We say that f attains a relative maximum value of $f(a)$ at $x = a$ if there is some interval (b, c) with $b < a < c$ and such that for all $x \in (b, c)$, $f(x) \leq f(a)$.

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We refer to these values as <u>extreme values</u> of f.

Fact

If f is a smooth continuous function and if f attains an extreme value at $x = a$, then the derivative f' crosses the input axis at $x = a$ and thus $f'(a) = 0$.

Example

Consider the function

$$f(x) = \begin{cases} x^3 - 12x + 17 & \text{if } x \leq 1, \\ x^2 + 5 & \text{if } x > 1. \end{cases}$$

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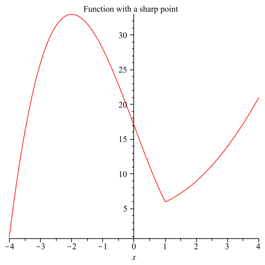
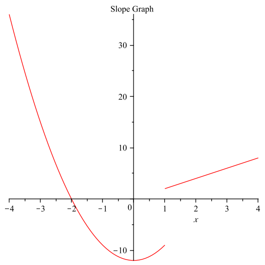
The graph of this function and its derivative are





Find all relative and absolute extreme points of f in the interval $[-4, 4]$.

Note

Relative extrema do not occur if the derivative touches the input axis but does not cross it. That is, $f'(a)$ may be zero even when f does not attain a relative extreme at $x = a$.

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Note

For a function f with input x, a relative extremum can occur at $x = c$ only if $f(c)$ exists. Furthermore,

1. A relative extremum exists where $f'(c) = 0$ and the graph of $f'(x)$ crosses the input axis at $x = c$.
2. A relative extremum can exist where $f(x)$ exists but $f'(x)$ does not.

Finding Extrema

To find the relative maxima and minima of a function f,

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2. Examine a graph of f to determine which of these input values correspond to relative maxima or relative minima.

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To find the absolute maximum and minimum of a function f on an interval $[a, b]$.

1. Find all relative extrema of f in the interval (as above).
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To find the absolute maximum and minimum of a continuous function f without a specified interval.

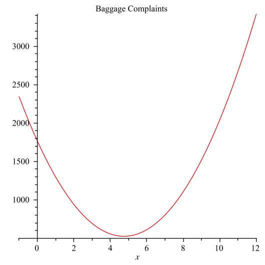
1. Find all relative extrema of f.
2. Determine the end behavior of the function in both directions. The absolute extrema either do not exist or are among the relative extrema.

Example

The number of consumer complaints to the US Department of Transportation about baggage on US airlines between 1989 and 2000 can be modeled by the function

$$B(x) = 55.15x^2 - 524.09x + 1768.65 \text{ complaints},$$

where x is the number of years after 1989.

1. The graph of the function is 

2. Find the relative and absolute maxima and minima on the interval $0 \leq x \leq 11$.