Recall: slope of the tangent line to $y = f(x)$ at $(a, f(a))$ is

$$m = \lim_{h \to 0} \left[ \frac{f(a+h) - f(a)}{h} \right]$$

Def.: The derivative of a function $f$ at a number $a$ is defined as

$$f'(a) = \lim_{h \to 0} \left[ \frac{f(a+h) - f(a)}{h} \right]$$

or

$$f'(a) = \lim_{x \to a} \left[ \frac{f(x) - f(a)}{x-a} \right]$$

Eg.: Let $f(x) = x^2 + 3x + 4$. Compute $f'(a)$. 
Tangent Line:

The tangent line to \( y = f(x) \) at \((a, f(a))\) is the line through \((a, f(a))\) with slope \( f'(a) \).
It has equation

\[
y - f(a) = f'(a) \ (x - a) \]

**eg.** Find the equation of the line tangent to \( y = x^2 + 2x + 1 \) at \((1, 4)\).
Rates of Change

If \( y = f(x) \), recall

\[ \Delta x = x_2 - x_1 \]
\[ \Delta y = f(x_2) - f(x_1) \]

and the instantaneous rate of change is

\[ \lim_{x_2 \to x_1} \left( \frac{\Delta y}{\Delta x} \right) = \lim_{x_2 \to x_1} \left[ \frac{f(x_2) - f(x_1)}{x_2 - x_1} \right] \]

\[ = f'(x_1) \]

Fact: The derivative \( f'(a) \) of \( f \) at \( a \) is the instantaneous rate of change of \( y = f(x) \) with \( x \) when \( x = a \).

(See example 5 pp. 130-131.)

eq The position of a particle is given by \( s(t) = \frac{1}{16} t^4 \) (\( t \) is in seconds; position is in m)

Find the velocity of the particle at time \( t = 2 \) s. What is the speed?