§3.14:

Recall: If \( x \) changes from \( x_1 \) to \( x_2 \) and if \( y = f(x) \) then we write

\[
\Delta x = x_2 - x_1, \quad (\text{the change in } x)
\]

\[
\Delta y = f(x_2) - f(x_1), \quad (\text{the change in } y)
\]

\[
\frac{\Delta y}{\Delta x} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad (\text{the average rate of change of } y \text{ w.r.t. } x \text{ in the interval } [x_1, x_2])
\]

\[
\frac{dy}{dx} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad (\text{the instantaneous rate of change in } y \text{ w.r.t. } x \text{ when } x = x_1)
\]

*Homework*: Interview at least two people who are employed in your major field of study and write a paragraph on why you need to know calculus for that field.
Suppose that a rod is made of a non homogeneous mixture of materials and that the mass of the rod between its left end pt. and the pt. $x$ meters to the right is given by

$$f(x) = \sqrt{x} \text{ kg}.$$

Mass between $x_1$ & $x_2$ is:

$$dm = f(x_2) - f(x_1)$$

Linear density at $x_1$ is:

$$\rho = \lim_{x_2 \to x_1} \left( \frac{f(x_2) - f(x_1)}{x_2 - x_1} \right)$$

$$= \frac{dm}{dx} \text{ or } f'(x) \text{ kg/m}$$

Eg. The linear density of the rod at $x=1.1$ is

$$f'(1.1) = \frac{1}{2 \sqrt{1.1}} \approx 0.4767 \text{ kg/m}$$
In a chemical reaction such as

$$2 \text{H}_2 + \text{O}_2 \rightarrow 2 \text{H}_2\text{O}$$

we have two reactants reacting to form a product.

$$\frac{A + B}{\text{reactants}} = \frac{C}{\text{product}}$$

We will denote the concentration in moles/liter of the reactants and product above at time $t$ as $A(t)$, $B(t)$, $C(t)$.

- Avg. rate of reaction $= \frac{AC}{dt} = \frac{C(t_2) - C(t_1)}{t_2 - t_1}$
- Instantaneous rate of reaction $= \frac{dC}{dt} = \lim_{t_2 \rightarrow t_1} \left[ \frac{dC}{dt} - C(t_1) \right]$  

It turns out that $A$ & $B$ decrease at the same rate as $C$ increases. That is,

$$\frac{dA}{dt} = -\frac{dB}{dt} = -\frac{dC}{dt}$$
The flow is fastest at the center of the vessel and slows as we move toward the vessel wall.

Let \( v \) denote the velocity of the flow. Then \( v \) decreases as \( r \) increases until \( v = 0 \) when \( r = R \).

- **Law of Laminar flow**
  \[
  v = \frac{p}{4 \pi \eta \ell} (R^2 - r^2)
  \]

\( p \) = diff of press on both ends  
\( \eta \) = blood viscosity.

*Note: If \( p \) and \( \ell \) are fixed then \( v \) is a function of \( r \) w/ domain \([0, R]\).*

avg. rate of change = \[
\frac{dv}{dr} = \frac{v(R) - v(r)}{R - r}
\]

velocity gradient = \[
\lim_{r \to R} \frac{v(R) - v(r)}{R - r} = v'(r)
\]

For small human arteries \( R = 0.008 \, \text{cm} \)

\( \eta = 0.02 \) 
\( \ell = 2 \, \text{cm} \)
How fast is the blood flowing when \( r = 0.002 \), \( r = 0.004 \)?

\[ v(0.002) = \]

\[ v(0.004) = \]

What is the velocity gradient when \( t = 0.004 \)?

\[ \frac{dv}{dt} = ? \]
Torriceili's Law

5000-gal tank
Water drains completely in 40 min.

\[ V(t) = 5000 \left(1 - \frac{t}{40}\right)^2 \text{ gal/min} \]
for \( t \in [0, 40] \)

\[ \text{velocity of water flow} \]

1. How fast is the water flowing at \( t = 5, 10, 20, 30 \) min?

\[ V(5) = \]

\[ V(10) = \]

\[ V(20) = \]

\[ V(30) = \]

2. When is it flowing fastest?

3. What is the instantaneous rate of change of the flow at time \( t \)?

\[ V(t) = 5000 \left(1 - \frac{t}{40} + \frac{t^2}{1600}\right) \]

\[ = 5000 - 250t + \frac{25}{8} t^2 \]

\[ V'(t) = \]