§3.8 Higher Derivatives.

The derivative of $f(x)$ is denoted $f'(x)$.

The derivative of $\frac{dy}{dx}$ is denoted $\frac{d^2y}{dx^2}$

* e.g. $f(x) = x^2 \sin(x)$

$\frac{d}{dx} f(x) = ?$
Note: If \( s(t) \) denotes the position of an object at time \( t \) then \( s'(t) \) gives the velocity of that object at time \( t \), \( s''(t) \) is the instantaneous rate of change of \( s'(t) \) (the velocity) w.r.t. time or the acceleration of the object.

\[
    v(t) = s'(t) \\
    a(t) = v'(t) = s''(t).
\]

Example: The position of a particle is given by \( s(t) = t^3 - 6t^2 + 9t \)

where \( t \) is measured in seconds and \( s \) is measured in meters.

a.) Find the acceleration at time \( t \).
    What is the acc. after 4s?

b.) When is the particle speeding up?
    When is it slowing down?
Higher derivatives.

We can of course also differentiate the derivative of \( f''(x) \). The derivative of \( f''(x) \) is denoted \( f'''(x) \).

The derivative of \( \frac{dy}{dx} \) is denoted \( \frac{d^2y}{dx^2} \).

In general, \( f^{(n)}(x) \) denotes the \( n^{th} \) derivative of \( f \).

**Example.** Let \( f(x) = x^3 + 3x^2 + 2x + 5 \)

Compute all derivatives of \( f \). 

**Example.** Compute all derivatives of \( g(x) = \frac{1}{x} = x^{-1} \).
eq. \quad \text{Find } y'' \text{ if } y \text{ is defined implicitly by } x^4 + y^4 = 16

eq. \quad \text{Find } f^{(23)}(x) \text{ if } f(x) = \sin(x)$. 