A ball is thrown upward with a speed of 48 ft/s from the edge of a cliff 432 ft above the ground. Find its height above the ground t seconds later. When does it reach its maximum height? When does it hit the ground?
Approximate the area under the curve \( y = x^2 \) from \( x=0 \) to \( x=1 \).

\[
R_H = \sum_{i=1}^{4} \text{area of } i^{th} \text{ rectangle}
\]

\[
L_H = \sum_{i=1}^{4} \text{area of } i^{th} \text{ rectangle}
\]
Compute the area in the first example.

Useful formula: \[ \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \]

\[ 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6} \]
More generally,

\[ y = f(x) \]

What is the area of \( S \)?

**Step 1:** Divide \( S \) into \( n \) strips

\[ S_1, S_2, S_3, \ldots, S_n \]

\[ a \leq x_1 < x_2 < \ldots < x_n \leq b \]

Area of \( S \) = \[ \sum_{i=1}^{n} \text{area of } S_n \]

**Width of a strip**

\[ \Delta x = \frac{b-a}{n} \]

*Note: We have subdivided \([a, b]\) into \( n \) subintervals \([x_0, x_1], [x_1, x_2], \ldots, [x_{n-1}, x_n] \).*
\[ x_0 = a \]
\[ x_1 = a + \Delta x \]
\[ x_2 = x_1 + \Delta x = a + 2\Delta x \]
\[ x_3 = x_2 + \Delta x = a + 3\Delta x \]
\[ \vdots \]
\[ x_i = a + i\Delta x \]
\[ \vdots \]
\[ x_n = a + n\Delta x = a + n \left( \frac{b-a}{n} \right) = b \]

Step 2: Approximate the area of \( S_i \) by the area of the rectangle of width \( \Delta x \) and height \( f(x_i) \)

\[ \text{Area (} S_i \text{)} \approx \Delta x \cdot f(x_i) \]

\[ \text{Area (} S \text{)} \approx \sum_{i=1}^{n} \Delta x \cdot f(x_i) \]

\[ \rightarrow R_n \]
Def: The area of the regions that lies under the graph of the continous function $f(x)$ is given by

$$A = \lim_{n \to \infty} R_n = \lim_{n \to \infty} \left[ \sum_{i=1}^{n} f(x_i) \Delta x \right]$$

Fact: We would get the same value of $A$ if we had used left end points, or in fact if we had taken the height of our rectangle to be $f(x_i^*)$ where $x_i^*$ is any point in $[x_{i-1}, x_i]$.

So:

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x \quad \text{(right endpoints)}$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_{i-1}) \Delta x \quad \text{(left endpoints)}$$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*) \Delta x$$
Example: Let \( A \) be the area of the region that lies under \( y = \cos(x) \) between \( x = 0 \) and \( x = b \) where \( 0 < b < \frac{\pi}{2} \).

a) Using right endpoints, find an expression for \( A \) as a limit.

b) Estimate the area when \( b = 2 \) by using 4 rectangles with height given by \( \cos(x^*) \) where \( x^* \) is the mid point.
The distance problem.

The velocity of an object in ft/sec is given at the following times:

<table>
<thead>
<tr>
<th>time</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>V</td>
<td>17</td>
<td>21</td>
<td>24</td>
<td>29</td>
<td>32</td>
<td>31</td>
<td>28</td>
</tr>
</tbody>
</table>

Approximate the distance travelled.

Recall

\[ \text{dist} = \text{vel} \times \text{time} \]

When \(0 \leq t \leq 5\)

\[ 17 \leq V \leq 21 \]

\[ \Rightarrow \text{dist} = 5 \times 17 = 85 \]

When \(5 \leq t \leq 10\)

\[ 21 \leq V \leq 24 \]

\[ \text{dist} = 21 \times 5 = 105 \]

Continuing we get that the total distance travelled from \(t=0\) to \(t=30\) is approximately

\[ \text{dist} = 5 \times 17 + 5 \times 21 + 5 \times 24 + 5 \times 29 + 5 \times 32 + 5 \times 31 \]

\[ = 5 \times (17 + 21 + 24 + 29 + 32 + 31) \]

\[ = 5 \times 154 = 770 \text{ ft} \]
In general if an object has velocity \( v(t) \). Then the distance travelled from \( t = a \) to \( t = b \) is given by

\[
\begin{align*}
d &= \lim_{n \to \infty} \sum_{i=1}^{n} v(t_{i-1}) \Delta t \\
&= \lim_{n \to \infty} \sum_{i=1}^{n} v(t_{i}) \Delta t
\end{align*}
\]

This is simply the area under \( y = v(t) \) from \( a \) to \( b \).