Question 12: Let \( f(x) = \frac{2x^2 - z}{x^2 - 4} \).

a) What is the domain of \( f(x) \)?
\[-\infty, -2, -2, 2, 2, \infty\]

b) What are the \( x \) - and \( y \) - intercepts?
\( x \)-intercepts: \( x = \pm 1 \)
y-intercept: \( y = \frac{1}{2} \)

c) Is \( f \) even, odd, or neither?
Even

d) What are the vertical asymptotes of \( f(x) \)?
Justify your answer using one-sided limits.
\[ x = \pm 2 \] \( \lim_{x \to 2^-} f(x) = -\infty \] \( \lim_{x \to 2^+} f(x) = \infty \]
\[ x = \pm 2 \] \( \lim_{x \to 2^-} f(x) = -\infty \] \( \lim_{x \to 2^+} f(x) = \infty \]

e) Identify the critical points and intervals of increase/decrease for \( f(x) \). Also indicate any local extremes.
Critical points: \( x = 0 \)
\( f \) is undefined at \( x = \pm 2 \) but so is \( f \) at \( x = 0 \) at \( x = 0 \).

f) Identify intervals of concavity and inflection points.
Inflection points: \( (2, 2), (2, 2) \)

\( f'' \) is undefined at \( x = 0 \) as is \( f' \).
\( f'' \) is never 0.

\( f'' \) changes from concave down to concave up at \( x = 0 \).

\( f'' \) has no points of inflection.

\( f'' \) is undefined at \( x = 0 \).

\( f'' \) is undefined at \( x = \pm 2 \).

\( f'' \) is undefined at \( x = 0 \).

\( f'' \) is undefined at \( x = \pm 2 \).

\( f'' \) is undefined at \( x = 0 \).

Thus, \( y = 2 \) is the only horizontal asymptote.

h) Sketch the graph of \( y = f(x) \).