MAT 106
Quiz #16
November 5, 2004

Name: Key

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

(1) Evaluate the following.
   a.) $\int \left( \frac{\sin(x)}{1 - \sin^2(x)} \right) dx = \int \frac{\sin(x)}{\cos(x)} dx = \int \tan(x) \sec(x) \, dx = \sec(x) + C$

   2 pt.'s

   b.) $\int \left( u^2 + 1 + \frac{1}{u} \right) du = \frac{1}{3} u^3 + u - \frac{1}{u} + C$

   2 pt.'s

   c.) $\int_0^4 (2v + 5)(3v - 1) \, dv = \int_0^4 (6v^2 + 13v - 5) \, dv = \left[ 2v^3 + \frac{13}{2} v^2 - 5v \right]_0^4$

   $= \left( 128 + 104 - 20 \right) - (0) = 212$

   3 pt.'s

(2) The velocity function of a particle in meters per second is given by $v(t) = 3t^2 - 4t$.
   a.) Find the displacement of the particle during the time interval $[0, 2]$.

   displacement $= \int_0^2 (3t^2 - 4t) \, dt = \left[ \frac{t^3}{3} - 2t^2 \right]_0^2 = (8 - 8) - (0) = 0$

   3 pt.'s

   b.) Find the total distance traveled by the particle during the time interval $[0, 2]$.

   $v(t) = 3t^2 - 4t = t (3t - 4)$

   $\begin{array}{cccc}
   v(t) & + & - & + \\
   (-\infty, 0) & (0, \frac{4}{3}) & (\frac{4}{3}, \infty) \\
   \end{array}$

   $\text{total dist. travelled} = \int_0^{\frac{4}{3}} |v(t)| \, dt = \int_0^{\frac{4}{3}} (3t^2 - 4t) \, dt + \int_{\frac{4}{3}}^2 (3t^2 - 4t) \, dt$

   $= \left[ \frac{t^3}{3} + \frac{2}{3} \right]_0^{\frac{4}{3}} + \left[ \frac{t^3}{3} - 2t^2 \right]_{\frac{4}{3}}^2$

   $= \left( \frac{4}{27} - 0 \right) + \left( 0 + \frac{2}{27} \right) = \frac{4}{27} \text{ m.}$