MAT 106
Quiz #2
September 15, 2004

Name:_____

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

(1) List the vertical asymptotes of the following function. Be sure to explain your answer.

$$f(x) = \frac{2x+3}{x-1}.$$

Solution: First, we note that as x gets close to 1 from the left that the numerator approaches 5 while the denominator approaches 0 and is negative. So,

$$\lim_{x\to 1^-} f(x) = -\infty.$$

As x approaches 1 from the right, the numerator approaches 5 while the denominator approaches 0 and is positive. Thus,

$$\lim_{x\to 1^+} f(x) = \infty.$$

So, by our definition of vertical asymptote in the book, we see that the line $x = 1$ is a vertical asymptote of $f(x)$.

(2) Use the squeeze theorem to compute the following limit.

$$\lim_{x\to 0}[x\sin(x)].$$

Solution: First, we recall that $-1 \leq \sin(x) \leq 1$. So, it follows that

$$-|x| \leq x\sin(x) \leq |x|.$$

Next, we compute

$$\lim_{x\to 0^-} |x| = \lim_{x\to 0^-} -x = 0, \quad \text{and} \quad \lim_{x\to 0^+} |x| = \lim_{x\to 0^+} x = 0.$$

Thus,

$$\lim_{x\to 0} |x| = 0.$$

Similarly, we compute

$$\lim_{x\to 0^-} -|x| = \lim_{x\to 0^-} x = 0, \quad \text{and} \quad \lim_{x\to 0^+} -|x| = \lim_{x\to 0^+} -x = 0.$$

Thus,

$$\lim_{x\to 0} -|x| = 0.$$

Since, $-|x| \leq x\sin(x) \leq |x|$ and since $\lim_{x\to 0} -|x| = 0 = \lim_{x\to 0} |x|$, it follows from the squeeze theorem that

$$\lim_{x\to 0}[x\sin(x)] = 0.$$