

MAT 106
Quiz #7 Key
17 September 2004

Name: _____

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

(1) Let $f(x) = \tan(\sqrt{x+5})$. Compute $f'(x)$.

$$\begin{aligned} f'(x) &= \sec^2(\sqrt{x+5}) \frac{d}{dx}(\sqrt{x+5}) \\ &= \sec^2(\sqrt{x+5}) \cdot \frac{1}{2}(x+5)^{-\frac{1}{2}} \\ &= \frac{\sec^2(\sqrt{x+5})}{2\sqrt{x+5}}. \end{aligned}$$

(2) Let $y^2 + xy + y = x^3 + 2x + 3$. Find y'' .

Implicitly differentiating, we obtain

$$\begin{aligned} 2yy' + y + xy' + y' &= 3x^2 + 2 \\ \Rightarrow y' &= \frac{3x^2 + 2 - y}{2y + x + 1}. \end{aligned}$$

Now implicitly differentiating again we obtain

$$\begin{aligned} y'' &= \frac{\frac{d}{dx}(3x^2+2-y) \cdot (2y+x+1) - \frac{d}{dx}(2y+x+1) \cdot (3x^2+2-y)}{(2y+x+1)^2} \\ &= \frac{(6x-y') \cdot (2y+x+1) - (2y'+1) \cdot (3x^2+2-y)}{(2y+x+1)^2} \\ &= \frac{\left(6x - \left(\frac{3x^2+2-y}{2y+x+1}\right)\right) \cdot (2y+x+1) - \left(2\left(\frac{3x^2+2-y}{2y+x+1}\right) + 1\right) \cdot (3x^2+2-y)}{(2y+x+1)^2}. \end{aligned}$$