Chapter 2 Collections.

Section 7: Lists

Def: A list is an ordered sequence of objects.

Example: $(1,2,3)$

$(1,1,2,4)$

$(1,2,5,\mathbb{Z},\gamma)$

Def: The length of a list is the number of objects in the list.

Example:

$(1,2,3)$ has length ______.

$(1,2,\mathbb{Z})$ has length ______.

$(1,1,1)$ has length ______.

$(\ )$ has length ______.

the empty list.

- **Def:** Two lists are considered to be equal provided that they have the same length and that the elements in corresponding positions are the same.

- **Example:**

$$(1,2,3) = (1,2,3)$$
$$(1,2,3) \neq (1,2,3,4)$$
$$(1,2,3) \neq (1,3,2)$$

- **Def:** Two element lists are called <u>ordered pairs</u>.

- **Counting:** How many 2 element lists whose elements are 1, 2 or 3 are there?

There are ______.

- **Fact:** The number of ordered pairs whose elements are chosen from $1, 2, \ldots, n$ is ________ .

- **Proof:**

- **Theorem 7.2:** (**Multiplication** Principle) Consider ordered pairs for which there are n choices for the first element and for each choice of the first element there are m choices for the second element. The number of such lists is ________ .

• <u>Example:</u> List all ordered pairs with the first element being 1, 2 or 3 and the second element being A or B.

There are ______ such lists.

• <u>Example:</u> Consider ordered pairs where the first element is 1, 2 or 3 and where if the 1st element is [redacted], the second is A or B. and if the 1st element is 2, then the second is C or D. and if the 1st element is 3, then the second is E or F. List all such ordered pairs.

There are ______.

Proof (of Multiplication Principle).

- Example: List all 2 element lists whose elements come from 1, 2, 3, or 4 where repeats are not allowed.

There are ______.

- **Corollary:** The number of ordered pairs without repeats whose elements are $1, 2, \ldots,$ or n is ______.

- **Pf:**

- **Longer lists:**
  Find all lists of length 3 whose elements are 1, 2 or 3.

There are ______.

Note: We have 3 columns in our chart. The lists in each column end in 1, 2 & 3 respectively.

The first two entries in each list are a 2-element list. We have arranged our chart so that each list in the same row begins with the same ordered pair (a, b, —)

Thus the number of rows is equal to the # of ordered pairs whose elements are 1, 2 or 3.

Thus,

$$\# \text{ 3-element lists from } 1,2,3$$
$$= 3 \times \# \text{ 2-element lists from } 1,2,3$$
$$= 3 \times 3^2 = 3 \cdot 9 = 27.$$

- <u>Generalization:</u> Consider 3-element lists where the elements are $1, 2, \ldots, n-1$ or n.

The number of such lists is ________.

Example: Find all 3 element lists without repeats whose elements are from 1, 2, 3 or 4.

There are ______.

Explanation:

3 element lists w/o repeats from 1,2,3,4

=

Generalization: Consider all 3-element lists w/o repeats whose elements are 1, 2, ... or n. There are ______ such lists.

(we are assuming of course that $n \geq 3$).

Extension of Mult. Principle:

Consider lists of length 3 where there are a choices for the first element and for each choice of the 1st element, there are b choices for the second element and for each choice of the 1st two elements, there are c choices for the third element. There are ______ such 3 element lists.

Example: A club with 10 members elects 4 officers (President, V.P., secratary, treasurer). How many possible outcomes for the election are there?

Theorem 7.6:

The number of length k lists from $1, 2, \ldots, n$ [redacted] is

$$\begin{cases} n^k & \text{if repeats are allowed.} \\ (n)_k & \text{if repeats are not allowed.} \end{cases}$$

Def:

$$(n)_k = \underbrace{n(n-1)(n-2)\cdots(n-k+1)}_{k\text{-factors.}}$$