Counting Subsets:

Let \( A = \{1, 2, 3\} \). List the subsets.

<table>
<thead>
<tr>
<th># elements</th>
<th>Subsets</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>( \emptyset )</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Total =

**Theorem 9.71**

Let \( A \) be a finite set. The number of subsets of \( A \) is \( 2^{#A} \).

**Proof:**
Definitions (Power Set)

Let \( A \) be a set. The power set of \( A \) is denoted \( 2^A \) and is defined to be the set of all subsets of \( A \).

Example:

Let \( A = \{1, 2\} \)

Then

\[
2^A = \{ \emptyset, \{1\}, \{2\}, \{1, 2\} \}.
\]

Corollary: If \( A \) is a finite set then

\[
|2^A| = 2^{|A|}.
\]

(This is a restatement of Theorem 9.7)
§ 2.10 Quantifiers.

There is: (Existential Quantifier)
Claims the existence of certain objects.

Example:
There is an integer \( x \) such that it is divisible by 2.

General:
There is \( x \in A \) such that \( P \).

Notation:
\[ \exists x \in A \text{ s.t. } P \]
\[ (\exists x \in \mathbb{Z} \text{ s.t. } 2 \mid x) \]

Proof Template 7:
To prove existential quantifiers \( \exists \) such as
\[ \exists x \in A \text{ s.t. } P, \]

Let \( x = \) (give an example)

Show that \( x \) satisfies \( P \).

Therefore \( x \) satisfies the required assertions.

Example:
\[ \exists x \in \mathbb{Z} \text{ s.t. } 2 \mid x. \]
Proof:
Let \( x = 2 \), then \( x = 2 \cdot 1 \)

Thus \( 2 \mid x. \)
For All (Universal Quantifier)

Example:
1. Every integer is either even or odd.
2. All integers are ___.
3. Each integer is ___.
4. Let \( x \in \mathbb{Z} \), then \( x \) is ___.

Notation:

\[ \forall x \in \mathbb{Z}, \ \text{x is even or odd} \]

General Form:

\[ \forall a \in A, \ \text{statements about a} \]

Proof template:
To prove \( \forall x \in A \), statements about \( a \).

Let \( x \in A \). Then, ___.

(prove all statements about \( a \) are true)

Example:

\( T = \{ x \in \mathbb{Z} : 10 \mid x^3 \} \)

\[ \forall x \in T \), \( 2 \mid x \].

Proof: Let \( x \in T \). ___
Negating Quantifiers:

\[ \neg (\exists x \in A \text{ st. } P) = \forall x \in A, \neg P, \]

\[ \neg (\forall x \in A, \leq) = \exists x \in A \text{ st. } \neg S. \]

Example:

1. Consider the statement:

   There is no integer that is both even and odd.

   Symbolically we write

   \[ \neg (\exists x \in \mathbb{Z} \text{ st. } x \text{ is even and } x \text{ is odd}) \]

2. Consider the statement:

   Not all integers are prime.

   Symbolically we write:
Consider the following statements. Are they true or false?

1. \( \forall x \in \mathbb{R}, \exists y \in \mathbb{R} \text{ s.t. } xy = 1 \)

2. \( \exists y \in \mathbb{R}, \forall x \in \mathbb{R}, xy = 1 \)

Prove or disprove.