• Counting Subsets:

Let $A = \{1, 2, 3\}$. List the subsets

# elements	Subsets	Number
0	$\emptyset$	1
1		
2		
3		
		Total =

Theorem 9.7:
Let A be a finite set. The number of subsets of A is $2^{|A|}$.

Proof:

Definition: (Power Set)

Let A be a set. The <u>power set</u> of A is denoted 2^A and is defined to be the set of all subsets of A.

Example:

Let $A = \{1, 2\}$

Then

$$2^A = \{ \qquad \qquad \qquad \}.$$

Corollary: If A is a finite set then

$$|2^A| = 2^{|A|}.$$

(This is a restatement of Theorem 9.7)

§ 2.10 Quantifiers.

- **There is:** (Existential Quantifier)
  Claims the existence of certain objects.

- **Example:**
  There is an integer x that is divisible by 2.

- **General:**
  There is $x \in A$ such that P.

- **Notation:**
  $\exists\, x \in A$ s.t. P

  ($\exists\, x \in \mathbb{Z}$ s.t. $2 \mid x$.)

- **Proof Template 7:**
  To prove existential quantifiers such as
  $\exists\, x \in A$ s.t. P,

  Let $x =$ __ (give an example)
  (show that x satisfies P).
  ⋮
  Therefore x satisfies the required assertions.

- **Example:** $\exists\, x \in \mathbb{Z}$ s.t. $2 \mid x$.

  Proof: Let $x = 2$. Then $x = 2 \cdot 1$
  Thus $2 \mid x$.

• For All: (Universal Quantifier)

• **Example:**
1.) Every integer is either even or odd.
2.) All integers are - - -
3.) Each integer is - - - -
4.) Let $x \in \mathbb{Z}$, then x is - - - - -

• *Notation:*

$$\forall x \in \mathbb{Z}, \ x \text{ is even or odd.}$$

• General Form:

$$\forall a \in A, \ \text{statements about } a$$

• Proof template 8: To prove $\forall x \in A$, statements about a:

Let $x \in A$. Then, ...
⋮
(prove all statements about a are true)

$\blacksquare$

• Example:

Let $T = \{x \in \mathbb{Z} : 10 \mid x\}$

$\forall x \in T, \ 2 \mid x$.

Proof: Let $x \in T$.

- Negating Quantifiers:

$$\neg(\exists x \in A \text{ s.t. } P) = \forall x \in A, \neg P.$$

$$\neg(\forall x \in A, S) = \exists x \in A \text{ s.t. } \neg S.$$

- Example:

(1) Consider the statement;

- There is no integer that is both even and odd.

Symbolically we write

$$\neg(\exists x \in \mathbb{Z} \text{ s.t. } x \text{ is even } \underline{\text{and}} \ x \text{ is odd})$$

$$=$$

(2) Consider the statements;

Not all integers are prime.

Symbolically we write;

$$=$$

Combinations:

Consider the following statements. Are they true or false?

___ 1. $\forall x \in \mathbb{R}, \exists y \in \mathbb{R}$ s.t. $xy = 1$

___ 2. $\exists y \in \mathbb{R}, \forall x \in \mathbb{R}, xy = 1.$

- Prove or disprove.