§19 Proof by contradiction,

- **Recall:**

$$A \longrightarrow B \quad \text{and} \quad \underbrace{\neg B \longrightarrow \neg A}_{\text{(contrapositive)}}$$

are logically equivalent.

- **Proof Template:** (Proof by contrapositive)

To prove "if A then B", instead prove "If not B then not A".

- **Proposition 19.1:**

Let R be an equivalence relation on a set A and let $a, b \in A$. If $\underbrace{a \not R b}_{A}$ then $\underbrace{[a] \cap [b] = \emptyset}_{B}$.

Proof: (Contrapositive)

We will show that $\neg B \Rightarrow \neg A$,

- **Proof by Contradiction:**

To prove "if A then B", we assume the conditions in A are met and <u>for the sake of contradiction</u> we assume that $\neg B$ is true. We then argue until we deduce a statement which is clearly false.

- **Prop 19.2:** (Justification for the above proof template).

$a \to b$ & $(a \wedge \neg b) \to FALSE$

are equivalent.

- **Pf:**

a	b	$a \to b$ *	$\neg b$	$a \wedge \neg b$	$a \wedge \neg b \to FALSE$ *
F	F	T	T	F	T
F	T	T	F	F	T
T	F	F	T	T	F
T	T	T	F	F	T

- **Prop 19.3:**

No integer is both even and odd.

($x \in \mathbb{Z} \Rightarrow x$ is not both even and odd.)

- **Pf:** We will assume for the sake of contradiction that $\exists x \in \mathbb{Z}$ s.t. x is both even and odd.

$\blacksquare$

- **Prop. 19.4:**

Let $a, b \in \mathbb{Z}$ with $a \neq 0$. Then there is at most one integer x with $ax + b = 0$.

- **Pf:**

$\blacksquare$