Name:_____

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

(1) Fill in the blank.

(2) a.) A ___theorem___ is a declarative statement which is always true without exception and for which there is a proof.

(2) b.) A ___conjecture___ is a declarative statement which we believe is true based on experiment or computational evidence but which we cannot yet prove.

(2) c.) ___Definitions___ are the building blocks of mathematics. They must be precise and unambiguous.

(4) (2) Prove that an integer x is odd if and only if $x+1$ is ~~odd~~ even

($\Rightarrow$): If $x \in \mathbb{Z}$ is odd

Then there is $c \in \mathbb{Z}$ such that
$$x = 2c+1$$
$$\Rightarrow x+1 = 2c+2 = 2(c+1)$$

Take $b = c+1$,

Then $b \in \mathbb{Z}$ and $x+1 = 2b$.

Thus $2 \mid x+1$

Therefore $x+1$ is even.

($\Leftarrow$): If $x+1$ is even. Then there is $c \in \mathbb{Z}$ such that $x+1 = 2c$,

So, $x = 2c-1 = 2(c-1)+1$

Take $b = c-1$. Then $b \in \mathbb{Z}$ and $x = 2b+1$.

Thus, x is odd.