MAT 119
Quiz #2
September 6, 2005

Name: ____________________________

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

(1) Fill in the blank.

a.) A \textit{theorem} is a declarative statement which is always true without exception and for which there is a proof.

b.) A \textit{conjecture} is a declarative statement which we believe is true based on experiment or computational evidence but which we cannot yet prove.

c.) \textit{Definitions} are the building blocks of mathematics. They must be precise and unambiguous.

(2) Prove that an integer \( x \) is odd if and only if \( x + 1 \) is even.

\((\Rightarrow)\): \( \neg x \in \mathbb{Z} \) is odd

Then there is \( c \in \mathbb{Z} \) such that \( x = 2c + 1 \)

\( \Rightarrow x + 1 = 2(c + 2) = 2(c + 1) \)

Take \( b = c + 1 \).

Then \( b \in \mathbb{Z} \) and \( x + 1 = 2b \).

Thus \( 2 \mid x + 1 \)

Therefore \( x + 1 \) is even.

\((\Leftarrow)\): \( \neg x + 1 \) is even. Then there is \( c \in \mathbb{Z} \) such that \( x + 1 = 2c \).

\( \Rightarrow x = 2c - 1 = 2(c - 1) + 1 \)

Take \( b = c - 1 \). Then \( b \in \mathbb{Z} \) and \( x = 2b + 1 \).

Thus, \( x \) is odd.