MAT 119
Quiz #3
September 13, 2005

Name: Key

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

(5 pt.'s) (1) Prove that if a and b are integers and $10|a$ and $6|b$ then $a+b$ is divisible by 2.

Suppose $a, b \in \mathbb{Z}$ and $10 \mid a$ and $6 \mid b$.
Then there are $c, d \in \mathbb{Z}$ such that

$$a = 10c \; ; \quad b = 6d$$

Thus,

$$a+b = 10c + 6d = 2(5c+3d)$$

Let $f = 5c+3d$.

then $f \in \mathbb{Z}$ and $a+b = 2f$

Thus $2 \mid a+b$ ∎

(5 pt.'s) (2) Prove that $x \leftrightarrow y$ is logically equivalent to $(x \wedge y) \vee (\neg x \wedge \neg y)$.

x	y	$(x \wedge y)$	$\neg x$	$\neg y$	$(\neg x \wedge \neg y)$	$(x \wedge y) \vee (\neg x \wedge \neg y)$ *	$x \leftrightarrow y$ *
F	F	F	T	T	T	T	T
F	T	F	T	F	F	F	F
T	F	F	F	T	F	F	F
T	T	T	F	F	F	T	T

Since $x \leftrightarrow y$ and $(x \wedge y) \vee (\neg x \wedge \neg y)$ have the same values in all possible situations, they are logically equivalent.