MAT 119  
Quiz #6  
October 11, 2005  

Name: Key  

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

(1) Suppose that \( R \) is a relation on a set \( A \). Prove that if \( R \) is symmetric then \( R = R^{-1} \).

(Note: It is actually true that \( R \) is symmetric if and only if \( R = R^{-1} \).)

Suppose that \( R \) is a symmetric relation on the set \( A \).

(i): Let \((x, y) \in R\) then \((y, x) \in R \) also.  
Thus \((x, y) \in R^{-1} \). Therefore, \( R \subseteq R^{-1} \).

(ii): Let \((x, y) \in R^{-1} \). Then \((y, x) \in R \).
Since \( R \) is sym, \((x, y) \in R \) also.  
Thus \( R^{-1} \subseteq R \).

Therefore \( R = R^{-1} \).

(2) Consider the equivalence relation

\[ R = \{ (1, 1), (2, 2), (3, 3), (4, 4), (3, 4), (4, 3), (2, 3), (3, 2), (2, 4), (4, 2) \} \]

on the set \( S = \{ 1, 2, 3, 4 \} \). Compute each of the following equivalence classes of \( R \):

\[ [1] = \{ 1 \} \]
\[ [2] = \{ 2, 3, 4 \} \]
\[ [3] = \{ 2, 4 \} \]
\[ [4] = \{ 2, 3, 4 \} \]