MAT 119
Quiz #6
October 11, 2005

Name: Key

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

(1) Suppose that R is a relation on a set A. Prove that if R is symmetric then $R = R^{-1}$. (**Note:** It is actually true that R is symmetric if and only if $R = R^{-1}$.)

Suppose that R is a symmetric relation on the set A.

($\subseteq$): Let $(x,y) \in R$ Then $(y,x) \in R$ also
Thus $(x,y) \in R^{-1}$. Therefore, $R \subseteq R^{-1}$.

($\supseteq$): Let $(x,y) \in R^{-1}$. Then $(y,x) \in R$.
Since R is sym., $(x,y) \in R$ also.
Thus $R^{-1} \subseteq R$.

Therefore $R = R^{-1}$.

(2) Consider the equivalence relation

$$R = \{(1,1),(2,2),(3,3),(4,4),(3,4),(4,3),(2,3),(3,2),(2,4),(4,2)\}$$

on the set $S = \{1,2,3,4\}$. Compute each of the following equivalence classes of R: $[1], [2], [3], [4]$.

$[1] = \{1\}$

$[2] = \{2, 3, 4\}$

$[3] = \{2, 3, 4\}$

$[4] = \{2, 3, 4\}$