**MAT 129**
**Lab #4**
**February 26, 2007**

(1) Write each of the following sentences using the quantifiers $\forall$ and $\exists$. Write their negation.
   a.) There is an integer whose square is 2.
   b.) Some integer is divisible by 7.
   c.) Every integer is a perfect cube.
   d.) All integers are divisible by 37.
   e.) There is an integer that when multiplied by any integer gives a result of 0.
   f.) Everybody loves somebody sometime.

(2) Let $A = \{a, b, c\}$.
   a.) Write down the power set $2^A$ of $A$.
   b.) How large is the power set of $2^A$? That is, compute $|2^{2^A}|$.
   c.) Write down 3 elements of $2^{2^A}$.

(3) There are actually some restrictions that we did not discuss on what sorts of collections of objects should be considered as sets. For instance let $M$ be the collection of all sets which do not contain themselves as elements. Now if $M$ is a set we can ask whether $M \in M$ or not. What do you think?

(4) Let $C = \{x \in \mathbb{Z} : x|c\}$ and let $D = \{x \in \mathbb{Z} : x|d\}$. Find a necessary and sufficient condition for $C \subset D$. (That is, find some statement $P$ about $c$ and $d$ such that $P$ is true if and only if $C \subset D$). Give a proof that your condition is correct.

(5) Suppose that $N > 1$ is an integer. Prove that the following integers are composite: $N!+2, N!+3, N!+4, \ldots, N!+N$. Use this observation to prove that the gaps between consecutive primes can be arbitrarily large.

(6) In this problem, we want to consider the decimal expansions of rational numbers.
   a.) Argue that the decimal expansion of any rational number must either be finite or must eventually become periodic.
   b.) Can you write the number $12.1347898989\bar{}$ as a fraction?
   c.) Argue that any number whose decimal expansion is either finite or is periodic is rational.
   d.) What can you say about the decimal expansion of $\sqrt{2}$?

(7) Suppose that $A, B, C$ and $D$ are sets. Show that

$$(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D).$$