

MAT 129
Lab #5
February 26, 2007

- (1) After picking up the tab for a fabulous dinner, the waiter named Bill gives you change consisting of 8 coins that appear to be identical. The next day you tell your friend David about wonderful evening you had. As it turned out, David had also dined in the same restaurant earlier in the evening and Bill was his waiter too. David, who has a reputation for being somewhat dishonest, brags about a joke he played on Bill. When it came time for David to pay his bill, he paid with cash, including one counterfeit coin. You begin to get nervous knowing that Bill also waited on you. David thinks this is a great joke since the counterfeit coin only differs from its real counterpart in weight. How can you determine if Bill passed on the counterfeit coin to you in just two weighings with a balance scale? How can you determine which (if any) coin is counterfeit in just three weighings?
- (2) Find the finishing order of a race involving contestants A, B, C, D, and E given that you overheard the following statements and know that the statements of those who finished first and second are false and the statements of all others are true:
 - a.) A: D was third
 - b.) B: E was not first
 - c.) C: I was not last
 - d.) D: C finished after B
 - e.) E: B was second
- (3) You are in charge of 7 children at a birthday party. To keep them occupied, you suggest a game of ring-around-the-rosy. The children are instructed to join hands and form a ring. One precocious 5 year-old asks you how many different ways there are to do this. What is your answer?
- (4) Let A , B and C be sets. Show that
$$|A \cup B| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|.$$
- (5) Suppose that A , B and C be sets. Show that
 - a.) $A \times (B - C) = (A \times B) - (A \times C)$.
 - b.) $A \times (B \Delta C) = (A \times B) \Delta (A \times C)$.