MAT 129
Lab #6
February 26, 2007

How to prove properties of a relation:
Let R be a relation on a set S. To show that R. is reflexive, we must show that xRx for all $x \in S$.
Since the statement "xRx for all $x \in S$" contains the universal quantifier "all", to prove this statement we must choose an arbitrary $x \in S$ and show that xRx.
Start out: Let $x \in S$. Then show: xRx. Since you're trying to show xRx, it is helpful to write out what xRx would mean (for example, in 1. below xRx means $x^2 \geq 0$) and see if you recognize this as being a true statement.
To show that R is symmetric, we must show that if $x, y \in S$ and xRy, then yRx.
Since the statement "if $x, y \in S$ and xRy, then yRx" is a conditional statement, to prove this statement, we should first try to prove this by the direct method.
Start out: Assume $x, y \in S$ and xRy. Then, show: yRx.
Write out what xRy means (for example, in 1. xRy means $xy \geq 0$). Since you're trying to show yRx, write out what yRx would mean (in 1., $yx \geq 0$) and try to see how this is a consequence of xRy.
To show that R is transitive, we must show that if $x, y, z \in S$ and xRy and yRz, then xRz.
Since the statement "if $x, y, z \in S$ and xRy and yRz, then xRz" is a conditional statement, to prove this statement, we should first try to prove this by the direct method.
Start out: Assume $x, y, z \in S$ and xRy and yRz. Then, show xRz.
Write out what xRy and yRz mean (in 1., xRy and yRz mean $xy \geq 0$ and $yz \geq 0$). Since you're trying to show xRz, write out what xRz would mean (in 1., $xz \geq 0$) and try to see how this is a consequence of xRy and yRz.

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For 1-7 below, do each of the following parts a-d.

a.) Prove or disprove: R is reflexive.
b.) Prove or disprove: R is symmetric.
c.) Prove or disprove: R is transitive.
d.) For 1-5 and 7: if R is an equivalence relation, describe the equivalence classes of R.

1.) Define R on $\mathbb{Z}$ by xRy if and only if $xy \geq 0$.
2.) Define R on $\mathbb{Q}$ by xRy if and only if $x - y \in \mathbb{Z}$.
3.) Define R on $\mathbb{Z}$ by xRy if and only if $x = 3y$.
4.) Define R on $\mathbb{Z} \times \mathbb{Z}$ by $(a, b)R(c, d)$ if and only if $a \geq c$.
5.) Define R on $\mathbb{Z}$ by xRy if and only if $x - y$ is a multiple of 9. Have you seen a relation like this one before?
6.) Let S be the set of all sets. Fix a set C in S. We'll use the set C to define a relation on S in the following way: Define R on S by ARB if and only if $A \cap C = B \cap C$ (So given two sets A and B, A is related to B if and only if $A \cap C = B \cap C$).
7.) Define a relation R on $\mathbb{Z} \times (\mathbb{Z} - 0)$ by $(a, b)R(c, d)$ if and only if $ad = bc$.

Challenge problem (1pt bonus): Think about why you might want to consider such pairs (a, b) in 7. as being equivalent. It might be helpful to think about (a, b) as defining a rational number $\frac{a}{b}$.