Ch 1:

§1 Jay. Read this on your own and do the exercise.

§2 Definitions.

Definitions are the building blocks of mathematics. They must be <u>precise</u> and <u>unambiguous</u>.

- Examples:

  Definition 2.1: An integer is <u>even</u> provided that it is divisible by 2.

What's wrong with this definition?

• Where to start?

We will assume familiarity with the integers:

$$\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, 4, \ldots\}$$

and with the operations:

$$+, \quad -, \quad *$$

and the relations

$$<, \leq, >, \geq, = .$$

• **Def 2.2:** We say that an integer a is <u>divisible</u> by an integer b provided that there is an integer c such that

$$a = bc.$$

If this is the case, then we also say:

1.) b <u>divides</u> a
2.) b is a <u>factor</u> of a
3.) b is a <u>divisor</u> of a
4.) <u>Notation</u>: $b \mid a$.

Example: True or False.

___ $3 \mid 12$.

___ $5 \mid 12$.

___ $-4 \mid 12$.

___ $2 \mid 2$.

Note: Now, Def. 2.1 of even makes sense and we can use it.

Is 12 even? Explain

Is 13 even?

- **Def. 2.4:** An integer a is called odd provided that there is an integer x such that
$$a = 2x + 1.$$
- **Example:** Is 13 odd?

- **Note:** It is not immediately obvious from our definitions that every number is either even or odd or that no number is both even and odd. These are "facts" that we must "prove".
- **Def 2.5:** An integer p is prime provided that $p > 1$ and the only positive divisors of p are 1 and p.

- **Example:** Which of the following are prime?

2	-2
6	-3
3	1
5	-1
9	11

- Why is 1 not prime?

- **Def 2.6:** A positive integer a is called composite provided that there is an integer b such that
  1. $1 < b < a$,
  2. $b \mid a$,

Example: Which of the following are composite

2
6
3
1
25

§3 Theorem.

A theorem is a declarative statement for which there is a proof. A statement which known to always be true without exception.

• **Example:**

1.) Non Theorem: It is raining.

2.) Non Theorem: In July Clemson is hot.

3) Non Theorem: When an object is dropped near the earth, it accellerates at a rate of $9.8\ m/s^2$.

4.) Theorem 3.1: (Pythagorus) If a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse, then

$$a^2 + b^2 = c^2$$

If - Then:

Many theorems in mathematics can be expressed in the form:

"If A then B."

Example: (The sum of two even integers is even.)

- **Theorem:** If ______________________

  then ______________________

Differences between Math & English.

Examples:

(1) If you don't finish your dinner then you will not get dessert.

English Interpretation: (2 expectations)

Math: (only one interpretation)

(2) If you clean your room, then I will give you \$10.

English:

Math:

- Suppose that the statement "If A then B" is true. Then, the math interpretation of this statement can be summarized as

A	B	Possible?
T	T	possible
T	F	impossible
F	T	possible
F	F	possible

- Notation:

	Read	If-then version
$A \Rightarrow B$	A implies B	If A then B
$A \Leftarrow B$	A is implied by B	If B then A

- **If and only If:**

"A if and only if B" means

1. $A \Rightarrow B$ (If A then B)
2. $B \Rightarrow A$. (If B then A)

Example:

Theorem: an integer x is even if and only if $x+1$ is odd.

Means 2 things:

1.)

2.)

- Notation: $A \Longleftrightarrow B$ means A if and only if B. (Also, A iff B).
- Summary of "if and only if":

A	B	Possible?
T	T	possible
T	F	impossible
F	T	impossible
F	F	possible

Supposing $A \Longleftrightarrow B$ is true

Truth Tables:

all possibilities

Variables:	A	B	C ...	Z	statements
	T	T	T ...	T	?
	T	F	F ---	T	?
	F ...			F	?

results T/F in given situation

Example:

A	B	$A \Rightarrow B$	$A \Leftrightarrow B$
T	T	T	
T	F	F	
F	T	T	
F	F	T	

• And, OR and Not:

A	B	A and B	A or B
T	T		
T	F		
F	T		
F	F		

A	not A
T	
F	

- <u>Other words for theorems:</u> (see p. 14)

  Result
  Fact
  Proposition
  Lemma
  Corollary
  Claim

- <u>Vacuous Truth.</u>

  Statements of the form "If A then B" for which A is never true are said to be vacuously true.

<u>example:</u>

If an integer x is both a perfect square and a prime then x is negative.