§6 Boolean Algebra.

- Operators: $\wedge$, $\vee$, $\neg$

A	B	$A \wedge B$	$A \vee B$	$\neg A$
T	T	T	T	F
T	F	F	T	F
F	T	F	T	T
F	F	F	F	T

- Example: Calculate the value of

$$(\neg \text{FALSE} \vee \neg \text{TRUE}) \wedge \text{TRUE}$$

- Def: Two boolean expressions are logically equivalent provided that they have the same values for all possible instances of their variables.

- **Prop 6.1:**

$$\neg(x \wedge y) = (\neg x) \vee (\neg y)$$

- **Proof:** In order to compare the above Boolean expressions in all possible situations, we construct a truth table

x	y	$x \wedge y$	$\neg(x \wedge y)$	$\neg x$	$\neg y$	$(\neg x) \vee (\neg y)$
F	F					
F	T					
T	F					
T	T					

- **Proof Template 4:** To show the equivalence of two Boolean expressions.
  1.) Construct a truth table showing the values of the two statements for all possible instances of their variables.
  2.) Check that the two expressions always have the same values.

- **Thm 6.2:**

1.) $x \wedge y = y \wedge x$; $x \vee y = y \vee x$.

2.) $x \wedge (y \wedge z) = (x \wedge y) \wedge z$; $(x \vee y) \vee z = x \vee (y \vee z)$.

3.) $x \wedge \text{TRUE} = x$; $x \vee \text{FALSE} = x$.

4.) $\neg(\neg x) = x$

5.) $x \wedge x = x$; $x \vee x = x$

6.) $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$;
$x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$.

7.) $x \wedge (\neg x) = \text{FALSE}$; $x \vee (\neg x) = \text{TRUE}$

8.) $\neg(x \wedge y) = (\neg x) \vee (\neg y)$;
$\neg(x \vee y) = (\neg x) \wedge (\neg y)$.

- **Proof:** We will prove #2 part 1. We will leave the proofs of the rest as an exercise. Make sure that you can do them on a test.

x	y	z	$(x \wedge y)$	$(y \wedge z)$	$(x \wedge y) \wedge z$	$x \wedge (y \wedge z)$
F	F	F				
F	F	T				
F	T	F				
F	T	T				
T	F	F				
T	F	T				
T	T	F				
T	T	T				

More Operators:

• Def: ($\rightarrow$ & $\leftrightarrow$).

x	y	$x \rightarrow y$	$x \leftrightarrow y$
F	F	T	T
F	T	T	F
T	F	F	F
T	T	T	T

• **Proposition 6.3:**

The expressions $x \rightarrow y$ and $(\neg x) \vee y$ are logically equivalent.

• **Proof:**

x	y	$x \rightarrow y$	$\neg x$	$\neg x \vee y$
F	F	T		
F	T	T		
T	F	F		
T	T	T		