§ 25 Composition.

Def: Let A, B, C be sets. Let
$f: A \to B$ and $g: B \to C$
Then we define a new function
$g \circ f : A \to C$ by
$$g \circ f(x) = g(f(x))$$

e.g.:
Let
$$A = \{1, 2, 3, 4\}$$
$$B = \{5, 6, 7\}$$
$$C = \{8, 9, 10, 11\}$$

$$f = \{(1,6), (2,6), (3,5), (4,7)\}$$
$$g = \{(5,8), (6,10), (7,11)\}$$
Then
$$g \circ f = \{$$

Note: $f \circ g$ and $g \circ f$ are frequently different functions. In fact, for both $f \circ g$ and $g \circ f$ to be defined, we need ______.

eg.: Let $A = \{0,1,2,3,4\}$

$f = \{(0,1), (1,2), (2,0), (3,0), (4,2)\}$

$g = \{(0,4), (1,1), (2,3), (3,0), (4,2)\}$

$f \circ g = \{$

$g \circ f = \{$

Prop 25.6: Let A, B, C, D be sets and let

$f: A \to B$
$g: B \to C$
$h: C \to D$

Then

$$(h \circ g) \circ f = h \circ (g \circ f).$$

Pf: We need to show

1.) $\operatorname{dom}((h \circ g) \circ f) = \operatorname{dom}(h \circ (g \circ f))$

2.) $\forall x \in \operatorname{dom}(h \circ (g \circ f))$ that
$h \circ (g \circ f)(x) = (h \circ g) \circ f(x)$.

Identity Function:

- **Def:** Let A be a set. The identity function id_A on A is defined by
$$id_A = \{(a,a) : a \in A\}$$
OR: $\forall a \in A \quad id_A(a) = a$.

- **Prop 25.8:** Let A, B be sets. Let $f: A \to B$. Then
$$f \circ id_A = id_B \circ f = f$$

- **Pf:**

- **Prop 25.9:** Let A, B be sets, & $f: A \to B$ is a bijection. Then
$$f \circ f^{-1} = id_B \quad \& \quad f^{-1} \circ f = id_A$$

- **Pf:** exercise.