

**MAT 129**  
**Quiz #3**  
**23 February 2005**

**Name:** \_\_\_\_\_

You may not use your notes. Please show all of your work. An answer without justification will receive little credit.

- (1) Consider the relation  $R$  defined on  $2^{\mathbb{Z}}$  by  $R = \{(A, B) : A, B \subseteq \mathbb{Z}; |A| = |B|\}$ . Prove that  $R$  is reflexive.

Let  $A \in 2^{\mathbb{Z}}$ . Then  $|A| = |A|$ . Thus,  $(A, A) \in R$ . So,  $R$  is reflexive.

- (2) Consider the relation  $R$  defined on  $2^{\mathbb{Z}}$  by  $R = \{(A, B) : A, B \subseteq \mathbb{Z}; |A| = |B|\}$ . Indicate whether  $R$  has each of the following properties and if not give the reason that it does not:

Reflexive: **Yes**.

Irreflexive: **No**. For example,  $(\{1, 2\}, \{1, 2\}) \in R$ .

Symmetric: **Yes**.

Antisymmetric: **No**. For example,  $(\{3, 4\}, \{1, 2\})$  and  $(\{1, 2\}, \{3, 4\}) \in R$ , but  $\{3, 4\} \neq \{1, 2\}$ .

Transitive: **Yes**.