

MTHSC 206 SECTION 12.2 –VECTORS

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GEOMETRIC DESCRIPTION OF VECTORS

DEFINITION

- 1 A vector $v \in \mathbb{R}^3$ is a pair of points $v = \vec{AB}$. The point A is said to be the initial point and the point B the terminal point.
- 2 The magnitude of $v = \vec{AB}$ is defined to be $|AB|$.
- 3 Two vectors are considered equal if they have the same direction (i.e. they lie on parallel lines and have the same orientation) and the same magnitude.
- 4 The zero vector denoted by 0 is the vector whose initial and terminal points are the same. This vector has magnitude 0 and has no associated direction.

DEFINITION (ADDITION)

If u and v are vectors positioned so that the initial point of v is at the terminal point of u , then $u + v$ is the vector with initial point the same as the initial point of u and terminal point the same as the terminal point of v .

NOTE

The parallelogram law assures us that $u + v = v + u$.

DEFINITION (SCALAR MULTIPLICATION)

If $c \in \mathbb{R}$ and $v \in \mathbb{R}^3$ then the scalar multiple cv of v by c is the vector whose length is $|c|$ times the length of v and whose direction is the same as v if $c > 0$ and is opposite of the direction of v if $c < 0$. If $c = 0$ or $v = 0$ then $cv = 0$.

NOTE

- 1 We define the negative of v as $(-1)v$.
- 2 We define the difference of two vectors as $u - v = u + (-1)v$.

ALGEBRAIC DESCRIPTION OF VECTORS

CONVENTION

- 1 Given a vector $a = \vec{AB}$, we can associate to it the pair (a_1, a_2) if we are in 2 dimensions or the 3-tuple (a_1, a_2, a_3) if we are in 3 dimensions where one can move from the initial point A to the terminal point B by moving a_1 units in the x direction, a_2 units in the y direction (and a_3 units in the z direction).
- 2 The pair or 3-tuple above is called an algebraic vector.
- 3 This completely characterizes the vector v .
- 4 So, we typically think of vectors as having their initial point at the origin and their terminal point at (a_1, a_2, a_3) .
- 5 We say that any geometric vector $v = \vec{AB}$ with associated algebraic vector $a = (a_1, a_2, a_3)$ is a representation of a .

FACT

Given the points $A = (x_1, y_1, z_1)$ and $B = (x_2, y_2, z_2)$, the algebraic vector a with geometric representation \vec{AB} is

$$a = (x_2 - x_1, y_2 - y_1, z_2 - z_1).$$

FACT

The magnitude or length of an algebraic vector a (or of any of its geometric representations) is given by

$$|a| = \begin{cases} \sqrt{a_1^2 + a_2^2} & \text{in 2 dimensions,} \\ \sqrt{a_1^2 + a_2^2 + a_3^2} & \text{in 3 dimensions.} \end{cases}$$

FACT

If $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$ and $c \in \mathbb{R}$, then

$$a + b = (a_1 + b_1, a_2 + b_2, a_3 + b_3),$$

$$a - b = (a_1 - b_1, a_2 - b_2, a_3 - b_3) \quad \text{and}$$

$$ca = (ca_1, ca_2, ca_3).$$

EXERCISE

Suppose that $a = (1, 2, -1)$ and $b = (-1, 3, 2)$. Find $|a|$, $a + b$ and $2a + 3b$.

PROPERTIES OF VECTORS

If a, b and c are vectors $d, e \in \mathbb{R}$, then,

① $a + b = b + a.$

② $(a + b) + c = a + (b + c).$

③ $a + 0 = a.$

④ $a + (-a) = 0.$

⑤ $d(a + b) = da + db.$

⑥ $(e + d)a = ea + da.$

⑦ $(ed)a = e(da).$

⑧ $(1)a = a.$

DEFINITION

The standard basis vectors are

$$i = (1, 0, 0), \quad j = (0, 1, 0) \quad \text{and} \quad k = (0, 0, 1).$$

FACT

$$(a_1, a_2, a_3) = a_1i + a_2j + a_3k.$$

EXERCISE

Suppose that $a = 2i - 3j + k$ and $b = i - j + 3k$. Express the vector $2a + b$ in terms of i, j and k .

DEFINITION

A unit vector is a vector whose length is 1.

EXAMPLE

i, j and k are unit vectors.

NORMALIZATION

If $0 \neq a \in \mathbb{R}^3$, then the unit vector which points in the same direction as a is given by $\frac{1}{|a|}a$.

EXERCISE

Suppose that a 100-lb weight hangs from two wires which are at 60° and 30° to the flat ceiling. Find the tension forces T_1 and T_2 in both wires and their magnitudes. (**Hint:** *Treat all forces as vectors in \mathbb{R}^2 expressed in terms of i and j .*)