MTHSC 206 Section 12.2 –Vectors

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4. The zero vector denoted by 0 is the vector whose initial and terminal points are the same. This vector has magnitude 0 and has no associated direction.

Definition (Addition)

If u and v are vectors positioned so that the initial point of v is at the terminal point of u, then $u + v$ is the vector with initial point the same as the initial point of u and terminal point the same as the terminal point of v.

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Note

The parallelogram law assures us that $u + v = v + u$.

Definition (Scalar multiplication)

If $c \in \mathbb{R}$ and $v \in \mathbb{R}^3$ then the scalar multiple cv of v by c is the vector whose length is $|c|$ times the length of v and whose direction is the same as v if $c > 0$ and is opposite of the direction of v if $c < 0$. If $c = 0$ or $v = 0$ then $cv = 0$.

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Note

1. We define the negative of v as $(-1)v$.
2. We define the difference of two vectors as $u - v = u + (-1)v$.

Convention

1. Given a vector $a = \vec{AB}$, we can associate to it the pair (a_1, a_2) if we are in 2 dimensions or the 3-tuple (a_1, a_2, a_3) if we are in 3 dimensions where one can move from the initial point A to the terminal point B by moving a_1 units in the x direction, a_2 units in the y direction (and a_3 units in the z direction).

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3. This completely characterizes the vector v.
4. So, we typically think of vectors as having their initial point at the origin and their terminal point at (a_1, a_2, a_3).
5. We say that any geometric vector $v = \vec{AB}$ with associated algebraic vector $a = (a_1, a_2, a_3)$ is a representation of a.

Fact

Given the points $A = (x_1, y_1, z_1)$ *and* $B = (x_2, y_2, z_2)$, *the algebraic vector* a *with geometric representation* $\vec{AB}$ *is*

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Fact

The magnitude or length of an algebraic vector a *(or of any of its geometric representations) is given by*

$$|a| = \begin{cases} \sqrt{a_1^2 + a_2^2} & \text{in 2 dimensions,} \\ \sqrt{a_1^2 + a_2^2 + a_3^2} & \text{in 3 dimensions.} \end{cases}$$

Fact

If $a = (a_1, a_2, a_3)$ *and* $b = (b_1, b_2, b_3)$ *and* $c \in \mathbb{R}$, *then*

$$\begin{aligned} a + b &= (a_1 + b_1, a_2 + b_2, a_3 + b_3), \\ a - b &= (a_1 - b_1, a_2 - b_2, a_3 - b_3) \quad \text{and} \\ ca &= (ca_1, ca_2, ca_3). \end{aligned}$$

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Exercise

Suppose that $a = (1, 2, -1)$ and $b = (-1, 3, 2)$. Find $|a|$, $a + b$ and $2a + 3b$.

Properties of Vectors

If a, b and c are vectors $d, e \in \mathbb{R}$, then,

1. $a + b = b + a$.
2. $(a + b) + c = a + (b + c)$.
3. $a + 0 = a$.
4. $a + (-a) = 0$.
5. $d(a + b) = da + db$.
6. $(e + d)a = ea + da$.
7. $(ed)a = e(da)$.
8. $(1)a = a$.

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Exercise

Suppose that $a = 2i - 3j + k$ and $b = i - j + 3k$. Express the vector $2a + b$ in terms of i, j and k.

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Normalization

If $0 \neq a \in \mathbb{R}^3$, then the unit vector which points in the same direction as a is given by $\frac{1}{|a|}a$.

Exercise

Suppose that a 100-lb weight hangs from two wires which are at 60° and 30° to the flat ceiling. Find the tension forces T_1 and T_2 in both wires and their magnitudes. *(**Hint:** Treat all forces as vectors in $\mathbb{R}^2$ expressed in terms of i and j.)*