# MTHSC 206 Section 12.6 – Cylinders and Quadric Surfaces

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## Note

In order to draw 3D pictures it is useful to compute the intersections of a surface with planes parallel to the coordinate planes. The resulting curves are called traces.

## Definition

A cylinder is a surface which consists of all lines which are parallel to a given line and which pass through a given plane curve.

## Example

Graph the following cylinders.

1. $z = y^2$
2. $x^2 + y^2 = 25.$
3. $x^2 + z^2 = 36.$

## Definition

A quadric surface is the graph of an equation of degree 2 in the variables $x$, $y$ and $z$.

## Note

The most general such equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0,$$

where $A, B, C, D, E, F, G, H, I, J \in \mathbb{R}$.
However, by translating (moving in the $x$-, $y$-, and $z$-directions) the surface can be assumed to have an equation of one of the following forms.

1. $Ax^2 + By^2 + Cz^2 + Iz = 0$,
2. $Ax^2 + By^2 + Cz^2 + J = 0$.

## Exercise

Graph the following surfaces.

1. Ellipsoid: $\frac{x^2}{25} + y^2 + \frac{z^2}{4} = 0$.
2. Elliptic Paraboloid: $z = x^2 + 9y^2$.
3. Hyperbolic Paraboloid: $z = x^2 - y^2$.
4. Cone: $z^2 = x^2 + y^2$.
5. Hyperboloid of One Sheet: $x^2 + y^2 - z^2 = 1$.
6. Hyperboloid of Two Sheets: $-x^2 - y^2 + z^2 = 1$.