

# MTHSC 206 SECTION 13.1 – VECTOR FUNCTIONS AND SPACE CURVES

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## DEFINITION

A vector valued function is a function of a real variable whose output is a vector.

That is, for each  $t \in \mathbb{R}$  there is a vector  $r(t) \in \mathbb{R}^3$ .

We usually write  $r(t) = (f(t), g(t), h(t))$  where  $f$ ,  $g$  and  $h$  are real valued functions called the components of  $r$ .

## EXAMPLE

$$r(t) = \left(1 + \frac{1}{t}, \frac{1}{t^2}, 2 + e^{-t}\right).$$

## DEFINITION

We define the limit of a vector valued function as follows. If  $r(t) = (f(t), g(t), h(t))$ , then we define

$$\lim_{t \rightarrow a} r(t) = \left( \lim_{t \rightarrow a} f(t), \lim_{t \rightarrow a} g(t), \lim_{t \rightarrow a} h(t) \right)$$

## EXAMPLE

Take  $r(t) = (1 + \frac{1}{t}, \frac{1}{t^2}, 2 + e^{-t})$  as above. Then,

$$\lim_{t \rightarrow \infty} r(t) = (1, 0, 2).$$

## DEFINITION

We say that a vector function  $r(t)$  is continuous at  $a$  if

$$\lim_{t \rightarrow a} r(t) = r(a).$$

## NOTE

This means that  $r(t)$  is continuous if and only if its components are all continuous.

## DEFINITION

Suppose that  $f$ ,  $g$  and  $h$  are real valued functions and that  $C = \{(f(t), g(t), h(t)) \mid t \in \mathbb{R}\}$ . We say that  $C$  is a space curve. The equations  $x = f(t)$ ,  $y = g(t)$ ,  $z = h(t)$  are called parametric equations for  $C$  and  $t$  is called a parameter.

# FROM ALGEBRAIC DESCRIPTIONS TO GEOMETRIC ONES

## EXAMPLE

Describe the curve defined by the vector function

$$r(t) = (1 + 2t, 3 - 7t, 2 + 8t).$$

## EXAMPLE

Sketch the curve whose vector equation is given by

$$r(t) = i \cos(t) + j \sin(t) + k.$$

# FROM GEOMETRIC DESCRIPTIONS TO ALGEBRAIC ONES

## EXAMPLE

Find a vector equation and parametric equations for the line segment that joins  $(1, 1, 0)$  to the point  $(2, 3, 5)$ .

## EXAMPLE

Find a vector function that represents the curve of intersection of the cylinder  $x^2 + y^2 = 25$  with the plane  $x + y - z = 1$ .