MTHSC 206 Section 14.1 – Functions of Several Variables

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DEFINITION

A <u>function of two variables</u> is a rule which assigns to each pair (x, y) of real numbers in a set D called the <u>domain</u> a unique real value denoted by $f(x, y)$.

The set of all values attained by f is the <u>range</u> of f.

EXAMPLE

The area of a rectangle is a function of its length ℓ and its width w. So, it can be thought of as a function of two variables $A(\ell, w) = \ell w$.

CONVENTION

If an explicit rule is given for a function of two or more variables and the domain is not specified, then the domain is understood to be the set of all possible inputs for which the explicit rule gives a well defined real number.

What is the domain of the function $f(x, y) = \ln(x^2 - y^2)$?

If a function arises in an application then the domain maybe determined by the application itself.

Suppose that $V(r, h) = \pi r^2 h$ is the volume of a cylindrical water tank with radius r and height h. What is the domain of V.

Find the domain and range of the function
$f(x, y) = \sqrt{16 - x^2 - y^2}$.

Definition

Suppose that $f(x, y)$ is a function with domain D. The graph of f is the set $\{(x, y, f(x, y)) \mid (x, y) \in D\}$.

Example

Draw the graph of the function $h(x, y) = 2x^2 + y^2$.

DEFINITION

The <u>level curves</u> of a function $f(x, y)$ are the curves in $\mathbb{R}^3$ with equations $f(x, y) = k$ where $k \in \mathbb{R}$ is a constant.

NOTE

The level curves of $f(x, y)$ are the traces of the graph of $f(x, y)$ in the plane $z = k$.

EXAMPLE

Compute the level curves of $f(x, y) = \sqrt{16 - x^2 - y^2}$.

DEFINITION

A <u>function of *n* variables</u> is a rule that assigns to each *n*-tuple $(x_1, \ldots, x_n)$ in some subset D or $\mathbb{R}^n$ a real number $f(x_1, x_2, \ldots, x_n)$.
The set D is the <u>domain</u> of f.

NOTE

1. The graph of a function of *n* variables naturally lives in $\mathbb{R}^{n+1}$ which is not easily represented when $n > 2$.

2. When $n = 3$, we can gain some insight by considering the level surfaces $f(x, y, z) = k$, where $k \in \mathbb{R}$ is constant.

3. When a function of *n* variables is given by a rule and the domain is not specified we follow the same convention as in the $n = 2$ case. That is, we take the domain to be the set of all values $\vec{x} \in \mathbb{R}^n$ for which the rule for $f(\vec{x})$ gives a well-defined real number.

EXAMPLE

Find the domain and range of the function
$f(x, y, z) = \ln(16 - x^2 - y^2 - z^2)$. Plot some of its level surfaces.