MTHSC 206 Section 16.1 – Vector Fields

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Example

Plot $F(x, y) = (-y, x) = -yi + xj$. Note that F assigns to each vector an orthogonal vector.

Definition

1. Suppose that $f(x, y)$ is a differentiable function on $\mathbb{R}^2$. Then ∇f is a vector field on $\mathbb{R}^2$ called a gradient field.
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Example

Consider the function $f(x, y) = x^2 + y^2$. Plot the gradient field for f and several contours of f.

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In the case that $F = \nabla f$, we say that f is a potential function for F.