MTHSC 206 Section 16.9 – The Divergence Theorem

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Definition

We say that a solid region in $\mathbb{R}^3$ is a simple solid region if it is of type 1, type 2 and type 3 (see 16.6).

The Divergence Theorem

Let E be a simple solid region and let $S = \partial E$ be the boundary of E with positive orientation. Let $F = [P, Q, R]$ be a vector field whose components have continuous partials on an open region containing E. Then

$$\int\int_S F \cdot \, d\mathbb{S} = \int\int\int_E \operatorname{div}(F) \, dV$$

Example

Find the flux of the vector field $F(x, y, z) = [z, y, x]$ over the unit sphere $x^2 + y^2 + z^2 = 1$.

Example

Evalueate $\int\int_S F \cdot \, d\mathbb{S}$, where $F(x, y, z) = [xy, y^2 + e^{xz^2}, \sin(xy)]$ and S is the surface of the region E bounded by the parabolic cylinder $z = 1 - x^2$, the xy-plane, the xzplane and the plane $y + z = 2$.

Note

We can apply the divergence theorem to other regions as well.

1. If $E = E_1 \cup E_2 \cup \cdots \cup E_k$ is the union of a finite number of simple solids, then the divergence theorem will still hold for E.
2. Suppose that S_1 is a closed surface lying inside the closed surface S_2 and the E is the region between the two. Then,

$$\int\int\int_E \mathrm{div}(F)\ \mathrm{dV} = \int\int_{S_2} F \cdot\ \mathrm{d}\mathbb{S} - \int\int_{S_1} F \cdot\ \mathrm{d}\mathbb{S}$$