MTHSC 206 Section 13.1 –Three dimensional coordinate systems

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Goal

We wish to generalize the familiar xy-plane to three dimensions in order to model the three dimensional space we live in. In order to do this we need to introduce some new ideas.

- We use the traditional x- and y-axes and add a third z-axis which is perpendicular to the xy-plane.
- The positive direction along the z-axis will be determined by the so called *right-hand rule*.
- To each point P in space we associate a 3-tuple (a, b, c).
- To arrive at the point P, we travel a units along the x-axis, b units in the direction parallel to the y-axis and c units in the direction parallel to the z-axis.

Note

There three planes determined by any two of the 3 axes are called the xy-plane, the xz-plane and the yz-plane.

Exercise

What solution spaces are determined by the following?

- $z = 1$.
- $x = y$.
- $x = y = z$.
- $x = 2$, $y = 1$, $z = 3$.

Definition (Distance Formula)

The distance $|P_1P_2|$ between two points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is given by

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Note

This formula follows from the Pythagorean theorem.

Exercise

Find the distance between $(2, 1, 3)$ and $(1, -1, 5)$.

Exercise

Give an equation whose solution set is the points on the surface of the sphere centered at the point (h, k, l) and whose radius is r.

Exercise

What region of $\mathbb{R}^3$ is determined by the following inequalities.

$$1 \leq x^2 + y^2 + z^2 \leq 4, \quad z \leq 0$$

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