

MTHSC 206 SECTION 13.6 – CYLINDERS AND QUADRIC SURFACES

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NOTE

In order to draw 3D pictures it is useful to compute the intersections of a surface with planes parallel to the coordinate planes. The resulting curves are called traces.

DEFINITION

A cylinder is a surface which consists of all lines which are parallel to a given line and which pass through a given plane curve.

EXAMPLE

Graph the following cylinders.

- ① $z = y^2$
- ② $x^2 + y^2 = 25$.
- ③ $x^2 + z^2 = 36$.

DEFINITION

A quadric surface is the graph of an equation of degree 2 in the variables x , y and z .

NOTE

The most general such equation is

$$Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0,$$

where $A, B, C, D, E, F, G, H, I, J \in \mathbb{R}$.

However, by translating (moving in the x -, y -, and z -directions) the surface can be assumed to have an equation of one of the following forms.

- 1 $Ax^2 + By^2 + Cz^2 + Iz = 0,$
- 2 $Ax^2 + By^2 + Cz^2 + J = 0.$

EXERCISE

Graph the following surfaces.

- ① Ellipsoid: $\frac{x^2}{25} + y^2 + \frac{z^2}{4} = 0$.
- ② Elliptic Paraboloid: $z = x^2 + 9y^2$.
- ③ Hyperbolic Paraboloid: $z = x^2 - y^2$.
- ④ Hyperboloid of One Sheet: $x^2 + y^2 - z^2 = 1$.
- ⑤ Hyperboloid of Two Sheets: $-x^2 - y^2 + z^2 = 1$.