

MTHSC 206 SECTION 13.6 – CYLINDERS AND QUADRIC SURFACES

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EXAMPLE

Graph the following cylinders.

- 1 $z = y^2$
- 2 $x^2 + y^2 = 25$.
- 3 $x^2 + z^2 = 36$.

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The most general such equation is

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where $A, B, C, D, E, F, G, H, I, J \in \mathbb{R}$.

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However, by translating (moving in the x -, y -, and z -directions) the surface can be assumed to have an equation of one of the following forms.

- 1 $Ax^2 + By^2 + Cz^2 + Iz = 0,$
- 2 $Ax^2 + By^2 + Cz^2 + J = 0.$

EXERCISE

Graph the following surfaces.

- 1 Ellipsoid: $\frac{x^2}{25} + y^2 + \frac{z^2}{4} = 0$.
- 2 Elliptic Paraboloid: $z = x^2 + 9y^2$.
- 3 Hyperbolic Paraboloid: $z = x^2 - y^2$.
- 4 Hyperboloid of One Sheet: $x^2 + y^2 - z^2 = 1$.
- 5 Hyperboloid of Two Sheets: $-x^2 - y^2 + z^2 = 1$.