MTHSC 206 Section 14.1 – Vector Functions and Space Curves

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Definition

A vector valued function is a function of a real variable whose output is a vector.
That is, for each $t \in \mathbb{R}$ there is a vector $r(t) \in \mathbb{R}^3$.
We usually write $r(t) = (f(t), g(t), h(t))$ where f, g and h are real valued functions called the components of r.

Example

$$r(t) = (1 + \frac{1}{t}, \frac{1}{t^2}, 2 + e^{-t}).$$

Definition

We define the limit of a vector valued function as follows. If $r(t) = (f(t), g(t), h(t))$, then we define

$$\lim_{t\to a} r(t) = \left(\lim_{t\to a} f(t), \lim_{t\to a} g(t), \lim_{t\to a} h(t)\right)$$

Example

Take $r(t) = (1 + \frac{1}{t}, \frac{1}{t^2}, 2 + e^{-t})$ as above. Then,

$$\lim_{t\to\infty} r(t) = (1, 0, 2).$$

Definition

We say that a vector function $r(t)$ is continuous at a if

$$\lim_{t \to a} r(t) = r(a).$$

Note

This means that $r(t)$ is continuous if and only if its components are all continuous.

Definition

Suppose that f, g and h are real valued functions and that $C = \{(f(t), g(t), h(t)) \mid t \in \mathbb{R}\}$. We say that C is a space curve. The equations $x = f(t),\ y = g(t),\ z = h(t)$ are called parametric equations for C and t is called a parameter.

From algebraic descriptions to geometric ones

Example

Describe the curve defined by the vector function

$$r(t) = (1 + 2t, 3 - 7t, 2 + 8t).$$

Example

Sketch the curve whose vector equation is given by

$$r(t) = i\cos(t) + j\sin(t) + k.$$

From geometric descriptions to algebraic ones

Example

Find a vector equation and parametric equations for the line segment that joins $(1, 1, 0)$ to the point $(2, 3, 5)$.

Example

Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 25$ with the plane $x + y - z = 1$.