MTHSC 206 Section 14.1 – Vector Functions and Space Curves

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Example

$$r(t) = (1 + \frac{1}{t}, \frac{1}{t^2}, 2 + e^{-t}).$$

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$$\lim_{t\to\infty} r(t) = (1, 0, 2).$$

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Suppose that f, g and h are real valued functions and that $C = \{(f(t), g(t), h(t)) \mid t \in \mathbb{R}\}$. We say that C is a <u>space curve</u>. The equations $x = f(t),\ y = g(t),\ z = h(t)$ are called <u>parametric equations</u> for C and t is called a <u>parameter</u>.

Example

Describe the curve defined by the vector function

$$r(t) = (1 + 2t, 3 - 7t, 2 + 8t).$$

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Sketch the curve whose vector equation is given by

$$r(t) = i\cos(t) + j\sin(t) + k.$$

Example

Find a vector equation and parametric equations for the line segment that joins $(1, 1, 0)$ to the point $(2, 3, 5)$.

From geometric descriptions to algebraic ones

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Find a vector equation and parametric equations for the line segment that joins $(1, 1, 0)$ to the point $(2, 3, 5)$.

Example

Find a vector function that represents the curve of intersection of the cylinder $x^2 + y^2 = 25$ with the plane $x + y - z = 1$.