MTHSC 206 Section 15.1 – Functions of Several Variables

Kevin James

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Convention

If an explicit rule is given for a function of two or more variables and the domain is not specified, then the domain is understood to be the set of all possible inputs for which the explicit rule gives a well defined real number.

Example

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Find the domain and range of the function $f(x, y) = \sqrt{16 - x^2 - y^2}$.

Definition

Suppose that $f(x, y)$ is a function with domain D. The graph of f is the set $\{(x, y, f(x, y)) \mid (x, y) \in D\}$.

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Draw the graph of the function $h(x, y) = 2x^2 + y^2$.

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Example

Compute the level curves of $f(x, y) = \sqrt{16 - x^2 - y^2}$.

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A function of n variables is a rule that assigns to each n-tuple $(x_1, \ldots, x_n)$ in some subset D or $\mathbb{R}^n$ a real number $f(x_1, x_2, \ldots, x_n)$.

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1. The graph of a function of n variables naturally lives in $\mathbb{R}^{n+1}$ which is not easily represented when $n > 2$.
2. When $n = 3$, we can gain some insight by considering the level surfaces $f(x, y, z) = k$, where $k \in \mathbb{R}$ is constant.
3. When a function of n variables is given by a rule and the domain is not specified we follow the same convention as in the $n = 2$ case. That is, we take the domain to be the set of all values $\vec{x} \in \mathbb{R}^n$ for which the rule for $f(\vec{x})$ gives a well-defined real number.

Example

Find the domain and range of the function $f(x, y, z) = \ln(16 - x^2 - y^2 - z^2)$. Plot some of its level surfaces.